Fourier Pricing Method by Adaptive Fourier Decomposition

by

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DECLARATION

The author declares that this thesis represents her own work with Professor QIAN Tao, the author’s supervisor. All the work is done under the supervision of Prof. QIAN during the period 2011-2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author’s knowledge, by anyone else.

JIA Wen-Yan
Option Pricing Method by Adaptive Fourier Decomposition

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Abstract

As presented in many papers, we have various methods to obtain option pricing formula, such as Carr and Madan (1999) approach and Lipton’s (2001). The primary conclusions have been reached with the help of Fourier transformation and complex analysis in these papers, and Fast Fourier Transform (FFT) algorithm makes them possible to be the desired numerical results. In this thesis Adaptive Fourier Decomposition (AFD) is applied to achieve the result instead of the direct use of FFT algorithm. In addition, AFD is better than Fourier Decomposition in aspect of convergence and frequency. We intend to implement AFD to decompose integrand function given by Lewis (2001) and reach some explicit formula solutions that can be used in theory. As an indispensable part, we give a more rigorous formulation of option pricing based on harmonic analysis.
Contents

1 Introduction 1

2 Fourier Transform Method and Lévy Processes 3
   2.1 Fourier Transform and its Properties 3
   2.2 Lévy process 5
   2.3 Pricing European Vanilla Option 7

3 Adaptive Fourier Decomposition 10

4 Using AFD to Option pricing formula 15

5 Related Work 21

6 Summary and Conclusion 24

Bibliography 25