Design, Modeling and Control of a Compliant Parallel XY Micro-motion Stage with Complete Decoupling Property

by

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Master of Science in Electromechanical Engineering

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A thesis submitted in partial fulfillment of the requirements for the degree of

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A novel compliant parallel XY micro-motion stage is proposed to fulfill positioning and manipulation with micro- or nanoscale precision. With a symmetric 4-PP structure, the XY stage has complete decoupling property. The stage is driven by piezoelectric actuators (PZT), and right-circular flexure hinge is adopted to convey the movement. Double four-bar flexure is chosen as the prismatic joint because of its better stiffness performance than double parallelogram flexure.

The compliance model of the mechanism is built using simplified compliance matrix method, and then the kinematics, workspace and stress are analyzed. Lagrange’s equation is employed to derive the dynamic model of the mechanism. The dimensions are optimized using particle swarm optimization (PSO) algorithm in order to maximize the natural frequencies. Finite element analysis (FEA) result indicates that the XY stage with optimal dimensions has a linear force-deflection relationship and ideal decoupling property. The first-mode natural frequency is as high as 720.52 Hz, and the stage has the potential to achieve a 105 μm × 105 μm square workspace. To cope with the hysteresis existing in the PZT, the control system is constructed by a proportional-integral-derivative (PID) feedback controller with a feed-forward hysteresis compensator based on Preisach model.

A prototype of the completely decoupled XY stage is fabricated with aluminum alloy AL7075-T6 using wire electric discharge machining (WEDM) technique. The static
test shows that the XY stage has a 19.2 μm × 18.8 μm rectangular workspace with coupling less than 5%. The numerical Preisach model of hysteresis is built according to the experimental data, and the control strategy is implemented by a personal computer (PC) with MATLAB software. With the closed-loop control system, the XY stage can complete positioning, tracking and contouring tasks with small error.
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NOMENCLATURE

\[ C^j_i \]: compliance matrix of \( i \) in coordinate \( j \)

\( C_i \): input compliance matrix

\( C_{oi} \): compliance matrix relating input force to output displacement

\( c_i \): compliance factor

\( C_O \): output compliance matrix

\( D \): actual displacement output of PZT

\( D_I \): input displacement

\( D_O \): output displacement

\( D_{nom} \): nominal maximum displacement output of PZT

\( E \): Young’s modulus

\( F_I \): input force

\( F_O \): external force on the mobile platform

\( f \): natural frequency

\( f_{pl} \): preload of the PZT

\( J \): Jacobian matrix

\( K \): equivalent stiffness matrix of the micro-motion stage

\( K_I \): input stiffness matrix

\( k_p \): stiffness of PZT

\( k_s \): stiffness of spring load on PZT

\( M \): equivalent mass matrix of the micro-motion stage

\( P^j_i \): skew-symmetric matrix for translation from coordinate \( i \) to \( j \)

\( \overline{P}^j_i \): translation matrix of compliance from coordinate \( i \) to \( j \)

\( R^j_i \): matrix for rotation from coordinate \( i \) to \( j \)

\( \overline{R}^j_i \): rotation matrix of compliance from coordinate \( i \) to \( j \)
$S_f$: safety factor

$T_i^j$: transformation matrix of compliance from coordinate $i$ to $j$

$u(t)$: input voltage to the PZT

$u_{PID}(t)$: control voltage from PID controller

$u_d(t)$: control voltage from the compensator

$w(\alpha, \beta)$: weighting function in Preisach model

$Y(\alpha_k, \beta_k)$: Preisach function of switching-value pairs $(\alpha_k, \beta_k)$

$y(t)$: output displacement of PZT

$\gamma_{\alpha \beta}$: hysteresis relay

$\sigma_y$: yield strength

$r, w, t, l_i, h_i$: dimensions of the micro-motion stage
LIST OF ABBREVIATIONS

AFM: atomic force microscope
DAQ: data acquisition
DOF: degree of freedom
FEA: finite element analysis
GA: genetic algorithm
MEMS: microelectromechanical systems
P: prismatic
PC: personal computer
PID: proportional-integral-derivative
PRB: pseudo rigid body
PSO: particle swarm optimization
PZT: piezoelectric actuators
R: revolute
S: spherical
SISO: single-input-single-output
SMA: shape memory alloy
U: universal
WEDM: wire electric discharge machining
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DEDICATION

The author wishes to dedicate this thesis to his parents.
CHAPTER 1: INTRODUCTION

1.1 COMPLIANT PARALLEL MECHANISM

A compliant mechanism is a flexible mechanism that transfers or transforms force, motion or energy through deformation of elastic members in the mechanism. Unlike traditional rigid-link mechanisms which consist of rigid links connected at movable joints, compliant mechanisms gain at least some of their mobility from the deflection of flexible members rather than from movable joints only [1]. There are two categories of compliant mechanisms: partially compliant mechanisms and fully compliant mechanisms. Fully compliant mechanisms have no traditional joints and all the motion is obtained from the deflection of compliant members, while partially compliant mechanisms contain one or more traditional kinematics pairs. According to their different deformation manners, fully compliant mechanisms can be divided into lumped and distributed compliant mechanism. The former obtains motion from elastic deformation in localized areas, i.e. flexure hinges. In the latter the compliance is distributed through a large portion of the structure. Most of existing designs belong to lumped compliant mechanisms.

Compared with conventional rigid mechanisms, compliant mechanisms have several advantages in particular applications. Compliant mechanisms have the potential to accomplish a specified task with a considerably less number of parts than with a rigid counterpart. Compliant mechanisms have fewer or no movable joints, such as pin and sliding joints, the wear, vibration, noise and need for lubrication are reduced as a result. Reducing the number of movable joints can also increase mechanism precision, because backlash may be reduced or eliminated, which is a critical factor in the design of high-precision instruments.

Compliant mechanisms can be designed to have specific force-deflection properties, or to tend to a particular position, e.g. a compliant robot end-effector which has a
constant output force regardless of the input displacement. Compliant mechanisms can easily store or transform energy which can be released at a later time or in a different manner.

Another advantage of compliant mechanisms is the ease of being miniaturized. The reduction in the total number of the parts is a significant advantage in the fabrication of micromechanisms, compliant mechanisms have a promising future in actuators, sensors and other microelectromechanical systems (MEMS).

A parallel manipulator is defined as a mechanism made up of an end-effector and a fixed base, which are linked together by at least two independent kinematic chains [2]. Actuation takes place through several actuators. A parallel manipulator in which the number of chains is strictly equal to the number of DOF of the end-effector are called fully parallel manipulator, which satisfies the formula developed by Gosselin: $p(n - 6) = -6$, where $p$ represents the number of chains and $n$ is the number of rigid bodies within a chain.

Possessing closed-loop kinematic chains, a parallel manipulator in general has much higher stiffness than a serial counterpart of an open-loop structure. In a serial robot, the inevitable hysteresis and off-axis flexibility accumulate along the kinematic chain. While in a parallel robot, the chain is usually short, simple and can thus be rigid against unwanted movement, the off-axis flexibility of a joint is also constrained by the effect of other chains. Errors in one chain’s positioning are averaged with others, rather than being accumulated. Hence this closed-loop stiffness makes the parallel manipulator have excellent performance in terms of high rigidity, high load carrying capacity and high accuracy.

Another advantage of the parallel manipulator is that the actuators may be centrally mounted on a single base, and the movement of the arm takes place through links and joints alone. This reduction in mass along the arm leads to a lighter arm, and thus lighter actuators and faster movements. The overall moment of inertia is also reduced,
which is an advantage for dynamics performance. Furthermore, the static actuators arrangement avoids moving cables which could cause friction and hysteresis.

Owing to the characteristics above, the compliant mechanism with a parallel kinematic structure is capable of positioning and manipulating with high speed and high accuracy in a limited workspace, while conventional mechanisms with motors, gears, and joints cannot meet the requirements mainly due to the backlash, hysteresis, clearance and friction of the joints, and the geometric and dimensional errors of the components. So far, most of the micro-motion stages or micromanipulators are designed based on compliant parallel mechanisms.

J. W. Ryu et al. [3] developed a XYθ micropositioning stage consisting of a monolithic flexure hinge-based mechanism with PZT actuators, the system has a total translational range of 41.5 μm and 47.8 μm along the X and Y axes respectively, and a maximum rotational range of 322.8 arcsec. W. J. Zhang et al. [4] designed a 3-RRR planar micro-motion stage using flexure hinges, and it has a workspace of 77.28 μm and 71.02 μm along the X and Y axes respectively, and 2.16 mrad around Z direction. F. Gao et al. [5] introduced a micromanipulator which adopts a 6-PSS compliant parallel mechanism with decoupling structure. A six-dimension controller consisting of a Stewart platform with a six-dimension force or moment sensor is used to control the robot. M. L. Culpepper et al. [6] developed a 6-DOF low-cost nano-manipulator using a planar compliant structure. The resolution is higher than 5 nm in a 100 nm × 100 nm × 100 nm work volume, and the open-loop errors are less than 0.2% of the full scale. Y. M. Li et al. [7] designed a compact 2-DOF micromanipulator with a workspace around 180 μm × 180 μm, and the resolution is several nanometers. W. Dong et al. [8] developed a 6-DOF precision compliant parallel positioner, which is dually driven by six piezoelectric motors and six piezoelectric ceramics. The positioner can provide a large workspace of 10 mm in three linear motion directions and 6-arc-degrees in three angular motion directions with high accuracy. J. Li et al. [9] proposed a micromanipulator with a 3-RUU
parallel structure. This mechanism has three pure translational DOF and can complete injection of cow oocyte successfully. X. Tang and I.-M. Chen [10] developed a XYZ flexure-based parallel mechanism, which has a motion range more than 1 mm in each direction. The cross-axis error and parasitic rotation are small due to the decoupling structure. Y. K. Yong et al. [11] proposed a flexure-based XY nanopositioning stage which has the ability to scan over a range of $25 \mu m \times 25 \mu m$ with high scanning speed. The stage has its first dominant mode at $2.7 \text{kHz}$, and the cross-coupling between the X and Y axes is low enough that single-input-single-output (SISO) control strategies can be utilized for tracking control.

All these micro-motion stages and micromanipulators adopt parallel configurations, and employ flexure joints other than traditional mechanical joints to achieve ultra-high accuracy. In these designs, the motion is conveyed by elastic deformation of flexure joints, which eliminates the problems such as friction, wear, backlash and lubrication. Actuators with high positioning resolution, such as electromagnetic actuators and piezoelectric actuators, are used to drive the mechanisms. In order to achieve high accuracy closed-loop control, some systems adopt displacement sensors with high solution to monitor the motion of the stage, such as capacitive sensors, laser interferometers and optical reflective sensors.

1.2 FLEXURE JOINTS

Flexible joints consist of one or more flexure hinges which are the most important components in the lumped compliant mechanisms. The compliant micromanipulator relies on the elastic deformation of flexure hinges to carry out mechanical tasks of transferring and transforming energy, force, and motion. A flexure hinge is a thin member that provides the relative rotation between two adjacent rigid members through bending, as shown in Fig. 1.1. Flexure hinges offer several advantages such as no friction losses, no need for lubrication, no hysteresis, compactness, capacity to be utilized in small-scale applications, ease of fabrication, virtually no assembly, and no required maintenance [12]. The performance of a compliant mechanism much
depends on its flexible joints.

Figure 1.1: A Flexure Hinge

![Figure 1.1: A Flexure Hinge](image)

Figure 1.2: Prismatic Flexible Joints

![Figure 1.2: Prismatic Flexible Joints](image)

Figure 1.3: Revolute Flexible Joints

![Figure 1.3: Revolute Flexible Joints](image)

According to the DOF possessed, flexible joints can be divided into two categories as follows:

1) One-DOF joints include prismatic (P) joints and revolute (R) joints. Most of the existing flexible P joints are based upon a parallelogram configuration. Their flexibility is derived from leaf hinges or notch hinges as illustrated in Fig. 1.2(a) and 1.2(b), respectively. The compliant P joint shown in Fig. 1.2(c) is a good choice as an actuation joint. The compound parallelogram configuration as shown in Fig. 1.2(d) can be adopted as P joints to gain a large motion range [13].

The flexible revolute (R) joints can be made into notch or leaf shapes. Four typical flexible notch joints are shown in Fig. 1.3, which are called right circular,
elliptical, right angle, and corner filleted hinges, respectively. Each style has its advantages and drawbacks. The right circular hinge is the so-called precision-oriented hinge since it provides precise rotation because their centers of rotation do not displace as much as other flexure hinges, while highly localized stress is experienced upon deflection and thus limiting its rotary range. The elliptical, right angle, and corner filleted hinges possess a more favorable stress distribution and are well suitable for large displacement applications [14], [15], but their accuracy is lower than right circular hinges. The elliptical profile is also more difficult to be manufactured.

2) Multi-DOF joints include universal (U) joints and spherical (S) joints which are illustrated in Fig. 1.4 and 1.5 respectively. A normal universal flexible joint is made up of two perpendicularly arranged R joints. The first flexible U joint in Fig. 1.4 possesses a compact architecture but is difficult to be machined. An offset distance exists between the two R hinges in the second one (see Fig. 1.4(b)), and the third type (see Fig. 1.4(c)) does not possess a compact architecture. Thus, the fourth one (see Fig. 1.4(d)) appears to be an ideal alternative, which can also protect the R joints from over-bent, but the rotary range is smaller.
Fig. 1.5 shows several 3-DOF spherical flexible joints. The first type is commonly utilized due to its high accuracy, although it is not strong and difficult to be manufactured. The second one is applied in large-displacement situation and has a poor accuracy. In addition, the contours of these two S joints can be made into all the profiles illustrated in Fig. 1.3 with the corresponding characteristics mentioned above. The third type S joint shown in Fig. 1.5(c) is built by adding another R joint on a flexible U joint described in Fig. 1.4(d), which has a better stiffness.

The precise modeling of the flexure hinges is critical to guarantee the positioning accuracy of the mechanism using flexure hinges [16]. Therefore, compliance/stiffness equations of flexure hinges are demanded to be as accurate as possible to reduce the accumulated modeling errors over a compliant mechanism. There have been many methods adopted to derive satisfactory compliance/stiffness equations of flexure hinges, including integration of the linear differential equations of a beam [17], Castigliano’s second theorem [12], inverse conformal mapping [18] and empirical equations formed from finite element analysis (FEA) results [19].

J. M. Paros and L. Weisbord [17] were the first researchers introducing the right circular flexure hinge, which becomes one of the most typical hinges nowadays. They formulated design equations, including both full and simplified forms, to calculate compliances of flexure hinges. Especially the following rotational compliance equation is widespread used:

\[
\frac{\theta_z}{M_z} = \frac{9\pi r^{1/2}}{2Ewt^{5/2}}
\]  

(1-1)

where \( \theta_z \) represents the angular displacement around z axis, \( M_z \) represents the
torque, $E$ is the Young modulus of the material, $r$, $w$, and $t$ are the geometric parameters of the flexure hinge depicted in Fig. 1.6.

Besides, Smith and Schotborgh obtained the compliance equations from FEA results [19], [20]. N. Lobontiu developed the compliance equations by Castigliano’s second theorem [12]. Wu and Zhou derived concise compliance equations based on Paros and Weisbord’s full equations [21]. Y. Tseytlin developed rotational compliance equations ($\theta_z/M_z$) for circular and elliptical flexure hinges using the inverse conformal mapping method [18]. By comparing various proposed compliance/stiffness equations with FEA results, Y. K. Yong suggested a guideline for designers to select the most suitable and accurate compliance equations for circular flexure hinge-based compliant mechanisms [22].

Compliance equations can be derived via the methods mentioned above for not only right circular flexure hinges but also other type hinges such as elliptical, right angle, corner filleted hinges, etc.

![Serial XY Stages from PI Company](image)

(a) A stacked XY stage  (b) A nested XY stage

Figure 1.7: Serial XY Stages from PI Company

1.3 FLEXURE-BASED MICRO-MOTION XY STAGE

Ultra-precision XY stage plays an important role in the field of micro- and nanotechnology. Owning the capabilities of positioning and motion with micro- or nanoscale precision, it finds broad applications, such as positioning of samples in an atomic force microscope (AFM), optical fiber alignment, single molecule experiment
in physics and biology, micromanipulation and microassembly. Most of these applications require micrometer or even nanometer accuracy, and the flexure hinge-based stage is one of the best choices available.

![Figure 1.8: Two Serial XY Stages](image)

The kinematic structure has a very large effect on the performance of a XY stage. There are two basic structures of XY stages, i.e. series and parallel. Most serial stages adopt a stacked or nested structure with two one-degree-of-freedom (1-DOF) prismatic stages. Fig. 1.7 shows two types of serial XY stage from PI company [23]. The stage in Fig. 1.7(a) consists of two stacked 1-DOF prismatic stages. This design is simple and modular but has a high center of gravity. The stage in Fig. 1.7(b) adopts
a nested structure, and thus has a lower center of gravity and somewhat better dynamics compared with stacked system. As shown in Fig. 1.8(a), N. G. Dagalakis et al. [24] proposed a XY stage which has two parallel sets of cantilever beam flexures for each direction, in order to reduce crosstalk in the X and Y translations and create more linear and independent motions. P. Gao [25] developed a serial micro-positioning system (see Fig. 1.8(b)) utilizing flexure hinges. The system is composed of two-grade amplifiers and symmetrical prismatic mechanisms. The motion ranges in the X and Y directions reach 45 μm and 40 μm respectively, and the displacement resolution is 0.020 μm and 0.018 μm. The first natural frequency for the dual direction is 525 Hz and 558 Hz.

Compared with parallel XY stages, serial XY stages are easier to design and have substantially decoupled DOF, while they have several disadvantages. Moving bulky actuators lead to high inertia and reduce the bandwidth of the axes that carry them, especially when the motion range is large. Moving cables would cause friction and hysteresis, which are sources of disturbances for nanoscale positioning. While in parallel stages, the actuators may be mounted on a fixed base. The closed-loop kinematic chains make the parallel manipulator have excellent performance in terms of high rigidity, high load carrying capacity and high accuracy. Hence parallel stages are preferable than serial ones.

So far, several flexure hinge-based XY stages with parallel kinematics have been proposed. The stages [7], [11] mentioned in Section 1.1 are two examples and their structures are illustrated in Fig. 1.9(a-b). Besides, Q. Yao et al. [26] developed a micro-positioning XY stage (see Fig. 1.9(c)) comprised of two serially-connected parallelogram four-bar linkage mechanisms, and it has a 87 μm × 87 μm square workspace with a resolution about 20 nm. K.-B. Choi and J. J. Lee [27] proposed a compliant XY stage based on a 4-PP (P stands for prismatic joint) flexure-joint structure as shown in Fig. 1.9(d), in which a double compound amplification mechanism is adopted to guide the linear motion and amplify the displacement.
Although the compliant parallel stage has the advantages as mentioned above, a challenge existing in the parallel mechanism is the input and output decoupling. The input decoupling can be defined as actuator isolation, which means each actuator would not suffer extra loads induced by the actuation of other actuators. The output decoupling means one actuator only drives the end-effector or mobile stage in one axial direction. Generally, input coupling would impose undesired load on the actuator and even damage it. Output coupling would lead to complex kinematic models, and thus make precise control difficult to be implemented. Hence completely decoupled parallel mechanisms which have both input and output decoupling properties are desirable to avoid these problems. In a completely decoupled XY stage, each actuator independently produces motion on the stage in each direction without affecting each other.

In the past, only a few completely decoupled XY stages have been proposed. S. Awtar
[28] developed a completely decoupled XY stage using double parallelogram beam flexures as shown in Fig. 1.10(a). The stage has a large motion range of $5 \text{ mm} \times 5 \text{ mm}$. The cross-axis coupling, motion loss and actuation isolation are better than 1%. Y. M. Li et al. [29] proposed a totally decoupled flexure-based XY micromanipulator (see Fig. 1.10(b)) by employing double parallelogram flexures and displacement amplifiers. The workspace of the micromanipulator is around $117 \mu\text{m} \times 117 \mu\text{m}$ with a maximum crosstalk of 1.5%, and the nature frequency is around 110.2 Hz. Both of them built the decoupled structures based on 4-PP parallel mechanisms, and adopted double parallelogram flexures to obtain prismatic motion.

![Figure 1.10: Two Completely Decoupled XY Stages](image)

1.4 CONTRIBUTIONS

Stiffness is important to XY stages since compliant parallel mechanisms designed with high stiffness possess good dynamic characteristics such as high natural frequencies, good repeatability and can perform fast precise motion [30]. In view of it, this thesis presents the development of a completely decoupled XY stage with good stiffness performance. The contributions of this thesis are specified as follows:

1) A novel compliant parallel micro-motion XY stage is proposed based on a 4-PP parallel mechanism with symmetric structure. Double four-bar flexure is selected as the P joint because of its better stiffness performance than double parallelogram
flexure. Chapter 2 presents the mechanism design.

2) Kinetostatic analysis is carried out to the proposed XY stage. The compliance model which indicates the force-displacement characteristics of the compliant mechanism is built. Chapter 3 presents the compliance analysis using compliance matrix method. The output and input compliance of the XY stage are obtained, and then the kinematics, workspace and stress are analyzed based on the compliance model.

3) Dynamic model of the proposed XY stage is established in Chapter 4. Lagrange’s equation is used to derive the general dynamic model under the assumption that the double four-bar flexures produce ideal prismatic motion. The natural frequencies can be calculated from the modal equation which describes undamped free vibration of the compliant system.

4) The dimensions of the XY stage are optimized with the purpose to maximize the natural frequencies in Chapter 5. Under the constraints caused by the performance of PZT, material properties, possibility of machining and permissible volume, particle swarm optimization (PSO) algorithm is utilized to derive the optimal dimensions. In Chapter 6, the finite element analysis (FEA) of the XY stage with optimal dimensions is carried out via ANSYS software, in order to validate the established compliance model and evaluate the decoupling performance of the XY stage. Modal analysis is implemented to evaluate the dynamic performance as well.

5) A PID feedback controller with a feed-forward hysteresis compensator is designed to achieve high motion accuracy. In Chapter 7, the hysteresis in the PZT is modeled using Preisach modeling method, and then a PID feedback controller is integrated with a feed-forward compensator based on the hysteresis model. In Chapter 8, a prototype of the designed XY stage is fabricated and the closed-loop control strategy is implemented to fulfill micro-positioning and tracking.
2.1 MECHANISM DESIGN

Pseudo rigid body mechanism synthesis is a major way to design flexure hinge-based compliant mechanisms, which essentially uses the well-developed kinematic synthesis of rigid-body mechanisms but replaces the conventional joints with appropriate flexure joints. Many compliant parallel mechanisms are designed in this way because it is simple and efficient, such as the 2-PP, 3-RRR, 3-PPP, 3-UPU, 3RUU, 6-SPS, etc., where P, R, U and S denote the revolute, prismatic, universal and spherical joints respectively. Several parallel mechanisms with two translational DOF are proposed as shown in Fig. 2.1. Among them, the simplest structure for a completely decoupled XY stage is the 2-PP parallel mechanism depicted in Fig. 2.1(c), which is an over-constrained structure. Each kinematic chain of the mechanism consists of two orthogonal prismatic joints. With an orthogonal arrangement of the two chains, decoupled X- and Y-translations can be obtained on the mobile platform as long as ideal P joints are used. This thesis chooses this structure as the kinematic scheme for the XY stage design.

![Figure 2.1: Parallel Mechanisms for XY Stages](image)

Employing ideal flexure P joints is a key point in the design since the P joint determines the main performance of the stage, such as stiffness, precision, load carrying capacity, and workspace. An ideal flexure P joint should have relatively
much lower stiffness in the working direction than other directions. Three types of flexure P joints are shown in Fig. 2.2, in which right circular flexure hinges are adopted since they possess the smallest center-shift when performing rotation, and thus are competent for high-precision tasks in limited spaces.

![Figure 2.2: Three Types of Flexure P Joints](image)

The first type shown in Fig. 2.2(a) is a conventional parallelogram flexure, it has a simple structure, but cross-axis error $e_y$ along the $y$ axis is generated at the same time that $F_x$ is exerted on the motion stage and forces it to move along the $x$ direction. To solve this problem, the double parallelogram flexure and double four-bar flexure are proposed. As shown in Fig. 2.2(b), the double parallelogram flexure can be considered as a serial connection of two parallelogram flexures. The secondary stage is mobile and the four links have the same length, so when $F_x$ is applied to the motion stage, the cross-axis error on the motion stage is compensated by the cross-axis error on the secondary stage. The motion range of this P joint is large, but it owns the drawbacks that generally exist in other serial kinematic chains, such as error accumulation and high inertia. The asymmetric structure would lead to non-uniform
thermal expansion. Moreover, the secondary stage would introduce a vibrational mode with low frequency [29]. The double four-bar flexure as illustrated in Fig. 2.2(c) is symmetric. It has better stiffness performance but smaller motion range than the double parallelogram flexure because the two parallelogram flexures are connected in parallel in this type of P joint. A rigid-body double four-bar mechanism cannot work since it has zero DOF, but in the compliant mechanism, the flexure hinge is not exactly a rotary joint because a small amount of axial elongation is allowable. Hence the motion stage can move along $x$ direction without cross-axis error if the displacement is extremely small compared with the dimensions of the links.

![Diagram of an Output Decoupled XY Stage](image)

**Figure 2.3: An Output Decoupled XY Stage**

With the purpose of designing a XY stage of high stiffness, the double four-bar flexure is adopted as the P joint. If the linear actuators are capable of producing prismatic motion with high precision, and have high stiffness to resist the shear and bending loads, a XY stage could be designed as shown in Fig. 2.3. The actuator is mounted on the ground and rigidly connected to the flexure P joint. With the actuators driving the P joints directly, decoupled XY translations can be obtained on the mobile platform. However, some linear actuators with high resolution are sensitive to shear...
forces or bending moments such as piezoelectric actuators (PZT). As shown in Fig. 2.3, if the actuator in Y direction drives the stage, the other actuator would suffer shear force $F$ and bending moment $M$ which could damage it.

Hence an input decoupler which can guide the prismatic translation and prevent the actuator from shear forces and bending moments is indispensable. Coincidently, the double four-bar is competent to be a decoupler due to its relatively much higher stiffness in the normal direction than in the working direction. So the previous decoupled XY stage is improved to be the completely decoupled stage as shown in Fig. 2.4.

![Figure 2.4: A Completely Decoupled XY Stage](image)

Another problem still existing is that a moment $M$ will be generated on one edge of the stage as shown in Fig. 2.4 when the Y direction actuator exerts a force on the stage. If the thickness $t_s$ of the edge or the rotating stiffness of the P joint is not high enough, unignorable rotation or bend would occur on the mobile platform and lead to
movement of poor accuracy. To eliminate this risk, further improvement is carried out by mirroring the design as shown in Fig. 2.5. Besides enhancement in the in-plane stiffness and motion accuracy of the stage, the out-of-plane stiffness is also significantly improved because the stage is supported from all sides. More importantly, symmetry makes the design more robust against manufacturing variations [31]. The effects of temperature gradient and disturbance on the structure are also reduced. In addition, the mobile platform is cut out in the center to reduce the mass.

Figure 2.5: A Completely Decoupled XY Stage with Symmetric Structure

2.2 MATERIAL AND ACTUATOR SELECTION

Although the double four-bar flexure P joint has high stiffness, high stress generated in the flexure hinges hinders it from getting a large motion range. To make up for this problem, the materials which allow great elastic deformation without material failure
are preferable. Hence the material with high ratio of yield strength to Young’s modulus ($\sigma_y/E$) [1] is selected to fabricate the mechanism in order to make the permitted workspace of the stage as large as possible. The mechanical properties of several common materials are listed in Table 2.1. Ti-6Al-4V has the highest value of $\sigma_y/E$, but the density is much higher than AL7075-T6. To achieve a low mass, the aluminum alloy AL7075-T6 is selected to fabricate the XY stage.

### Table 2.1: Mechanical Properties of Several Common Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield strength ($\sigma_y$ MPa)</th>
<th>Young’s modulus ($E$ GPa)</th>
<th>Density ($\rho$ kg/m$^3$)</th>
<th>$\sigma_y/E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V</td>
<td>880</td>
<td>113.8</td>
<td>$4.43 \times 10^3$</td>
<td>0.0077</td>
</tr>
<tr>
<td>AL7075-T6</td>
<td>503</td>
<td>71.7</td>
<td>$2.81 \times 10^3$</td>
<td>0.007</td>
</tr>
<tr>
<td>ASTM A514</td>
<td>690</td>
<td>205</td>
<td>$7.85 \times 10^3$</td>
<td>0.0034</td>
</tr>
</tbody>
</table>

Figure 2.6: A Monolithic Completely Decoupled XY Stage

Conventional actuators such as motors and voice coils are not suitable for high-precision positioning systems because of friction, backlash and clearance in the
bearings, thermal deformation caused by coil, etc., which prevent the actuators from achieving high precision and stable performance, and they usually have a large volume. Shape memory alloys (SMA), magnetostrictive materials and piezoelectric ceramics are often used in micro- and nanopositioning systems. SMA actuators can deliver high forces over long distances but their cycling frequency is low since the temperature of the material is difficult to control. Magnetostrictive materials have high force and strain capability, but they require more power than piezoelectric materials and are more expensive than other smart materials. Hence piezoelectric actuator (PZT) is selected to drive the stage, since it possesses the properties of small size, nanoscale displacement resolution, high stiffness, large force output and fast frequency response. The XY stage can be manufactured monolithically by wire electric discharge machining (WEDM) technique and assembled with the PZTs as shown in Fig. 2.6.
In this chapter, we analyze the kinetostatic properties of the designed XY stage including compliance, kinematic, workspace and stress, with the purpose to understand the basic attributes of the micro-motion stage. Since the links have much higher stiffness than the flexure hinges, their deformation is negligible after suitable material is chosen. All the motion of the micro-motion stage is considered to be obtained from elastic deformation of the flexure hinges while the links are regarded as rigid bodies. Hence the modeling of the right circular flexure hinges is the primary problem to be tackled in the compliance analysis.

Figure 3.1: A Right Circular Flexure Hinge and Its PRB Model

There are various approaches to model a flexure hinge-based mechanism. The most frequently used one is pseudo rigid body (PRB) method [32]. In this method, a right circular flexure hinge is comprehensively represented as an ideal rotary joint with a torsional spring as illustrated in Fig. 3.1, and then classical analysis methods of rigid-body mechanism can be applied to study the compliant mechanism, including geometric vector method for kinematics analysis, virtual work principle for statics analysis, etc. PRB method is simple and can generate analytical solutions, but it only considers the compliance of flexure hinges in their working direction (i.e. rotational direction) without taking other directions into account, which leads to deficient models. Nonlinear modeling can be utilized for accurate and complete compliance analysis. The theoretical equations for compliance and stress calculation are derived by integration of linear differential equations in the hinge’s actual cross-section contour. However, nonlinear modeling is not straightforward with numerous integral
operations. Hence, the compliance matrix method is employed in this thesis, because it can deal with full compliance of compliant mechanisms with rapid calculation. In this method, the Hooke’s law applies since the translational and rotational displacements are small in the compliant XY stage.

Based on the compliance model, the workspace is analyzed and constraint equations for dimensions are derived under the consideration of output reduction in PZTs, stress due to the deformation of flexure hinges and buckling phenomenon which would be induced by compressive load.

3.1 COMPLIANCE MATRIX METHOD

Yoshihiko applied the matrix method to kinematic analysis of a translational 3-DOF compliant parallel mechanism for an instance of general compliant mechanisms [33]. The matrix method he presented is well applicable to compliant mechanisms with notched hinges, and it needs fewer nodes of matrices to calculate a compliance matrix than conventional finite element method, which leads to high calculation efficiency.

In this method, a flexure hinge is treated as having six DOF. Fig. 3.2 shows a flexure hinge and the coordinates at two different points. When an external force $F_i$ is exerted at point $O_i$, the displacement $D_i$ at that point is calculated by

$$
D_i = \begin{pmatrix}
\delta_x \\
\delta_y \\
\delta_z \\
\theta_x \\
\theta_y \\
\theta_z \\
\end{pmatrix} = \begin{pmatrix}
c_1 & 0 & 0 & 0 & 0 & -c_3 \\
0 & c_5 & 0 & 0 & 0 & 0 \\
0 & 0 & c_2 & c_4 & 0 & 0 \\
0 & 0 & c_4 & c_6 & 0 & 0 \\
0 & 0 & 0 & 0 & c_8 & 0 \\
-c_3 & 0 & 0 & 0 & 0 & c_7 \\
\end{pmatrix} \begin{pmatrix}
f_x \\
f_y \\
f_z \\
x \\
y \\
z \\
\end{pmatrix} = C_i F_i \quad (3-1)
$$

---

Figure 3.2: A Flexure Hinge with Coordinates
where \( f_n \) and \( \delta_n \) are the force and translational displacement in \( n \)-axis respectively, 
\( M_n \) and \( \theta_n \) are the moment and rotational displacement around \( n \)-axis respectively, 
and \( C_i \) is the compliant matrix that represents the compliance of the flexure in its 
local coordinate \( O_i-xyz \). The compliant factor \( c_i (i = 1, 2, \ldots, 10) \) can be calculated 
for different types of hinges by corresponding equations derived via various methods 
mentioned in Section 1.2.

The force \( F_j \) applied to point \( O_j \) can be transferred from coordinate \( O_j-xyz \) to 
another coordinate \( O_i-xyz \) by

\[
F_i = \begin{bmatrix} (R_i^j)^T & 0 \\ 0 & (P_i^j)^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} F_j \tag{3-2}
\]

where \( R_i^j \) is the rotation matrix of coordinate \( O_j-xyz \) with respect to \( O_i-xyz \), \( P_i^j \) represents the skew-symmetric operator for the translation vector \( \overrightarrow{O_iO_j} = [p_x, p_y, p_z]^T \)
expressed in coordinate \( O_i-xyz \), and \( I \) is an identity matrix.

\[
P_i^j = \begin{bmatrix} 0 & -p_x & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3-3}
\]

The displacement \( D_i \) at point \( O_i \) induced by force \( F_i \) can be transferred from 
coordinate \( O_i-xyz \) to coordinate \( O_j-xyz \) by

\[
D_j = \begin{bmatrix} R_i^j & 0 \\ 0 & R_i^j \end{bmatrix} \begin{bmatrix} I \\ (P_i^j)^T \end{bmatrix} D_i \tag{3-4}
\]

Combining Eq. (3-1), (3-2) and (3-4) leads to the transformation of compliance \( C_i \)
from its local coordinate \( O_i-xyz \) to coordinate \( O_j-xyz \).

\[
C_j = T_i^j C_i (T_i^j)^T \tag{3-5}
\]

The \( 6 \times 6 \) transformation matrix \( T_i^j \) takes on the following form

\[
T_i^j = \overrightarrow{R_i^j P_i^j} = \begin{bmatrix} R_i^j & 0 \\ 0 & R_i^j \end{bmatrix} \begin{bmatrix} I \\ (P_i^j)^T \end{bmatrix} = \begin{bmatrix} R_i^j & R_i^j (P_i^j)^T \\ 0 & R_i^j \end{bmatrix} \tag{3-6}
\]

where \( \overrightarrow{R_i^j} \) represents the rotation of the compliance and \( \overrightarrow{P_i^j} \) represents the translation.

As the serial chain shown in Fig. 3.3(a), when a force \( F \) is applied to the end of the
chain, the induced deflection \( X \) at point \( O \) is contributed by each flexure hinge involved in the serial chain. Assume that the compliance of each flexure hinge in its local frame is \( C_i \), the displacement value \( X \) can be derived by the superposition of the deflection at the end due to the elastic deformation of each hinge.

\[
X = \sum_{i=1}^{k} T_i^O X_i = \sum_{i=1}^{k} T_i^O C_i F_i = \sum_{i=1}^{k} T_i^O C_i (T_i^O)^T F = CF
\]  
\( (3-7) \)

where \( T_i^O \) represents the compliance transformation matrix from the local coordinate of hinge \( i \) to point \( O \) and \( X_i \) is the displacement of hinge \( i \).

\[
C = \sum_{i=1}^{k} T_i^O C_i (T_i^O)^T = T^* C^* (T^*)^T
\]  
\( (3-8) \)

As the parallel structure shown in Fig. 3.3(b), when a force \( F \) is applied to the platform, because the platform is a rigid body, the equilibriums of displacement at point \( O \) lead to the following equation:

\[
X = T_i^O X_i
\]  
\( (3-11) \)

where \( X \) represents the displacement at point \( O \), \( X_i \) is the displacement at the end
of limb $i$, and $T_i^O$ represents the compliance transformation matrix from the end of limb $i$ to point $O$. The force $F$ applied at the platform is distributed to the ends of each limb, so the equilibriums of force lead to the following equation:

$$F = \sum_{i=1}^{k} (T_i^O)^{-T} F_i \quad (3-12)$$

where $F_i$ is the force at the end of limb $i$.

According to the Hook’s law,

$$F = \sum_{i=1}^{k} (T_i^O)^{-T} F_i = \sum_{i=1}^{k} (T_i^O)^{-T} C_i^{-1} X_i = \sum_{i=1}^{k} (T_i^O)^{-T} C_i^{-1} (T_i^O)^{-1} X = C^{-1} X \quad (3-13)$$

where $C_i$ is the compliance of limb $i$, which can be derived by Eq. (3-8). Hence, we get the compliance $C$ of the parallel structure in coordinate $O$-xyz.

$$C = \left( \sum_{i=1}^{k} (T_i^O)^{-T} C_i^{-1} (T_i^O)^{-1} \right)^{-1} = \left( \overline{T} \overline{K} \overline{T}^T \right)^{-1} = \left( \sum_{i=1}^{k} (T_i^O) C_i (T_i^O)^T \right)^{-1} \quad (3-14)$$

where

$$\overline{T} = \begin{bmatrix} (T_1^O)^{-T} & (T_2^O)^{-T} & \cdots & (T_k^O)^{-T} \end{bmatrix} \quad (3-15)$$

$$\overline{K} = \text{diag}(C_1^{-1}, C_2^{-1}, \ldots, C_k^{-1}) \quad (3-16)$$

Figure 3.4: A Flexure Hinge and Its 3-DOF Model

### 3.2 COMPLIANCE MODEL

To simplify the compliance model, the flexure hinges in the XY stage are considered to have three DOF as shown in Fig. 3.4(b) due to the fact that the XY stage is planar, and thick enough to obtain much higher out-of-plane stiffness than in-plane stiffness. The full $6 \times 6$ compliance matrix for the flexure hinge is substituted by a simplified $3 \times 3$ compliance matrix, in which only the translational compliance along $x$ and $y$
axes and the rotational compliance around z axis are taken into account. The simplified transformation matrix is extracted from the full transformation matrix Eq. (3-6).

As shown in Fig. 3.4(a), when an external force $\mathbf{F} = [f_x \ f_y \ m_z]^T$ is exerted on point $O_i$ of the flexure hinge, the displacement $\mathbf{D} = [d_x \ d_y \ \theta_z]^T$ of that point in its local coordinate $O_i$-xy can be calculated by

$$
\mathbf{D} = \begin{bmatrix} c_1 & 0 & -c_3 \\ 0 & c_5 & 0 \\ -c_3 & 0 & c_7 \end{bmatrix} \mathbf{F} = C_i \mathbf{F}
$$

(3-17)

Under the guidance of literature [22], the equations for calculation of compliance factors $c_i$ are chosen as follows:

$$
c_1 = \frac{d_x}{f_x} = \frac{3}{4Ew(2r+t)} \left\{ 2(2 + \pi)r + \pi t + \frac{(2r + t)\sqrt{t(4r + t)}}{\sqrt{t^5(4r + t)^5}} \right. \\
\times \left[ -80r^4 + 24r^3t + 8(3 + 2\pi)r^2t^2 + 4(1 + 2\pi)rt^3 \\
+ \pi t^4 \right] + \frac{8r^3(44r^2 + 28rt + 5t^2)}{t^2(4r + t)^2} \right. \\
- \frac{8(2r + t)^4(-6r^2 + 4rt + t^2)}{\sqrt{t^5(4r + t)^5}} \times \left( \arctan \sqrt{1 + \frac{4r}{t}} \right) \\
(3-18)
$$

$$
c_3 = -\frac{d_x}{m_z} = c_7 \times r
$$

(3-19)

$$
c_5 = \frac{d_y}{f_y} = \frac{1}{Eb} \left[ -2 \tan^{-1} \frac{\gamma - \beta}{\sqrt{1 - (1 + \beta + \gamma)^2}} \\
+ \frac{2(1 + \beta)}{\sqrt{2\beta + \beta^2}} \tan^{-1} \left\{ \frac{2 + \beta}{\beta} \times \frac{\gamma - \beta}{\sqrt{1 - (1 + \beta - \gamma)^2}} \right\} \right]
$$

(3-20)

$$
c_7 = \frac{\theta_z}{m_z} = \left( \frac{Ebt^2}{12} \right) \left[ -0.0089 + 1.3556 \left( \frac{t}{2r} \right) - 0.5227 \left( \frac{t}{2r} \right)^2 \right]^{-1}
$$

(3-21)

where $\beta = t/2r$, and $\gamma = 1 + \beta$.

The compliance matrix is transferred from its local coordinate $O_i$-xy to target coordinate $O_j$-xy by
The simplified $3 \times 3$ transformation matrix $T_i^j$ takes on the following form

$$T_i^j = R_i^j \overrightarrow{P}^j = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & r_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & p_y \\ 0 & 1 & -p_x \\ 0 & 0 & 1 \end{bmatrix}$$

(3-23)

where $r_{ij}$ is the entry in the $i$-th row and $j$-th column of the rotation matrix for the rotation from $O_i$-xy to $O_j$-xy, and $[P_x \ P_y]^T$ is vector $\overrightarrow{O_iO_j}$ in coordinate $O_i$-xy.

### 3.2.1 Compliance of Double Four-Bar Flexure

As shown in Fig. 3.5, the transverse P joint in limb $A$ of the XY stage is taken as an example. When link $g$ is fixed base and link $o$ is mobile platform, the compliance of chain $a$ in coordinate $A$-xy can be derived as

$$C_a^A = C_1^A + C_2^A = T_1^A C_i (T_i^A)^T + T_2^A C_i (T_i^A)^T$$

(3-24)

where $C_i^A$ represents the compliance of flexure hinge $i$ in coordinate $A$-xy, $C_i$ is the compliance of flexure hinge $i$ in its local coordinate, and $T_i^A$ is the compliance transformation matrix from local coordinate of flexure hinge $i$ to coordinate $A$-xy ($i = 1, 2$).

Chains $b$ and $a$ are symmetric with respect to $y$ axis, so the compliance of chain $b$ in coordinate $A$-xy can be obtained by rotating the compliance of chain $a$ at $180^\circ$ around $y$ axis.
\[ C^A_b = \overline{R}_y(\pi)C^A_a \left( \overline{R}_y(\pi) \right)^T \]  \hspace{1cm} (3-25)

According to the top-down symmetry, the compliances of chain \( c \) and \( d \) in coordinate \( A-xy \) are obtained:

\[ C^A_c = \overline{R}_x(\pi)C^A_a \left( \overline{R}_x(\pi) \right)^T \]  \hspace{1cm} (3-26)

\[ C^A_d = \overline{R}_x(\pi)C^A_b \left( \overline{R}_x(\pi) \right)^T \]  \hspace{1cm} (3-27)

Chains \( a, b, c \) and \( d \) are connected to the mobile platform \( o \) in parallel, therefore the compliance of this transverse P joint in coordinate \( A-xy \) is derived as

\[ C^A_{\text{tran}} = \left( (C^A_d)^{-1} + (C^A_b)^{-1} + (C^A_c)^{-1} + (C^A_d)^{-1} \right)^{-1} \]  \hspace{1cm} (3-28)

In the same way, the compliance of the vertical P joint in coordinate \( A-xy \) is derived.

\[ C^A_{\text{vert}} = \left( (C^A_b)^{-1} + (C^A_d)^{-1} + (C^A_d)^{-1} + (C^A_d)^{-1} \right)^{-1} \]  \hspace{1cm} (3-29)

Figure 3.6: The Compliant Parallel XY Stage

3.2.2 OUTPUT COMPLIANCE

As shown in Fig. 3.6, when an external force \( F_o = [f_{ox} \quad f_{oy} \quad M_{oz}]^T \) is exerted on the mobile platform at point \( O \), a displacement \( D_o = [d_{ox} \quad d_{oy} \quad \theta_{oz}]^T \) is
generated on it. The relationship between $F_o$ and $D_o$ can be expressed by

$$D_o = C_o F_o$$  \hspace{0.5cm} (3-30)$$

where $C_o$ is defined as the output compliance.

Owing to the double symmetric property of the micro-motion stage, limb $A$ is picked out for the analysis. As shown in Fig. 3.5, the limb is constructed by series connection of a transverse P joint and a vertical P joint. Hence the compliance of the whole limb $A$ is obtained by adding the compliances of the two P joints together.

$$C_A = C_{vert}^A + C_{tran}^A$$  \hspace{0.5cm} (3-31)$$

Then the compliance is transferred from coordinate $A$-xy to the target coordinate $O$-xy on the mobile platform.

$$C_A^O = T_A^O C_A (T_A^O)^T$$  \hspace{0.5cm} (3-32)$$

We can derive the compliance matrices of kinematic limbs $B$, $C$ and $D$ by rotating the compliance of limb $A$ at $90^\circ$, $180^\circ$ and $270^\circ$ around $z$ axis, respectively.

$$C_B^O = R_x(\pi/2)C_A^O (R_x(\pi/2))^T$$  \hspace{0.5cm} (3-33)$$

$$C_C^O = R_x(\pi)C_A^O (R_x(\pi))^T$$  \hspace{0.5cm} (3-34)$$

$$C_D^O = R_x(3\pi/2)C_A^O (R_x(3\pi/2))^T$$  \hspace{0.5cm} (3-35)$$

Therefore, the output compliance of the whole parallel mechanism in coordinate $O$-xy is derived as

$$C_o = (((C_A^O)^{-1} + (C_B^O)^{-1} + (C_C^O)^{-1} + (C_D^O)^{-1})^{-1}$$  \hspace{0.5cm} (3-36)$$

### 3.2.3 Input Compliance

As shown in Fig. 3.6, when an input force $F_i = [f_{ix} \ f_{iy}]^T$ is exerted at the two input ends, an input displacement $D_i = [d_{ix} \ d_{iy}]^T$ is generated there. The relationship between $F_i$ and $D_i$ can be expressed by

$$D_i = C_i F_i$$  \hspace{0.5cm} (3-37)$$

where $C_i$ is defined as the input compliance.

The spring model of the micro-motion stage (see Fig. 3.7) is built to derive the input
compliance. To calculate the compliance of driving point $E$, the compliance of kinematic limbs $B$, $C$ and $D$, the compliance of the transverse and vertical P joints in limb $A$ are transferred to $E$-xy.

$$
C_B^E = T_B^E C_B^D (T_B^E)^T, \quad C_C^E = T_B^E C_C^D (T_B^E)^T, \quad C_D^E = T_B^E C_D^D (T_B^E)^T
$$

(3-38)

$$
C_{\text{trans}}^E = T_A^E C_{\text{trans}}^A (T_A^E)^T, \quad C_{\text{vert}}^E = T_A^E C_{\text{vert}}^A (T_A^E)^T
$$

(3-39)

Then these three limbs are connected to the transverse P joint of limb $A$ in series, and the compliance is calculated by

$$
C_{BCD-\text{trans}}^E = C_{BCD}^E + C_{\text{trans}}^E
$$

(3-41)

Finally the vertical P joint is parallely connected to the part consisting of limbs $B$, $C$, $D$ and the transverse P joint. Hence we obtain the compliance of driving point $E$ as

$$
C_E = ((C_{BCD-\text{trans}}^E)^{-1} + (C_C^E)^{-1})^{-1}
$$

(3-42)

Because of symmetry, the compliance of driving point $F$ is derived by rotating $C_E$ at $90^\circ$ around $z$ axis

$$
C_F = R_z(\pi/2)C_E \left( R_z(\pi/2) \right)^T
$$

(3-43)

Hence we can obtain the input compliance of the XY stage.
\[ C_l = \text{diag}(C_F(1,1), C_E(2,2)) \]  \hspace{1cm} (3-44)

Obviously, \( C_F(1,1) = C_E(2,2) \) and \( C_l \) is a scalar matrix since the XY stage owns a decoupling and symmetric structure. \( K_I = C_I^{-1} \) is defined as input stiffness.

3.3 KINEMATICS

The relationship between the input force \( F_I = [f_{ix} \quad f_{iy}]^T \) and output displacement \( D_O = [d_{ox} \quad d_{oy}]^T \) can be expressed by

\[
D_O = C_{oi} F_I
\]

where

\[
C_{oi} = \begin{bmatrix}
d_{ox}/f_{ix} & 0 \\
0 & d_{oy}/f_{iy}
\end{bmatrix}
\]  \hspace{1cm} (3-45)

Figure 3.8: The Simplified Spring Model of the Micro-motion Stage

To derive \( d_{oy}/f_{iy} \), the spring model is decomposed as shown in Fig. 3.8. When \( f_{iy} \) is applied to point \( E \), equations can be set up according to the equilibrium of force and displacement.

\[
f_{iy} = f_{iy1} + f_{iy2}
\]  \hspace{1cm} (3-47)

\[
y_E = \left( C_{BCD}^E(2,2) + C_{\text{tran}}^E(2,2) \right) f_{iy1} = C_{\text{vert}}^E(2,2) f_{iy2}
\]  \hspace{1cm} (3-48)

Substituting Eq. (3-48) into (3-47) leads to

\[
f_{iy1} = \frac{C_{\text{vert}}^E(2,2)}{C_{\text{vert}}^E(2,2) + C_{BCD}^E(2,2) + C_{\text{tran}}^E(2,2)} f_{iy}
\]  \hspace{1cm} (3-49)
The relationship between the displacement $d_{oy}$ and the force $f_{iy1}$ is written as

$$d_{oy} = C_{BCD}^{E}(2,2)f_{iy1} = C_{BCD}^{E}(2,2)f_{iy1} \quad (3-50)$$

Substituting Eq. (3-49) into (3-50), we can get

$$\frac{d_{oy}}{f_{iy}} = \frac{C_{BCD}^{E}(2,2)C_{vert}^{E}(2,2)}{C_{vert}^{E}(2,2) + C_{BCD}^{E}(2,2) + C_{tran}^{E}(2,2)} \quad (3-51)$$

Because of the symmetric structure, $C_{OI}$ is a scalar matrix.

$$d_{ox}/f_{ix} = d_{oy}/f_{iy} \quad (3-52)$$

From Eq. (3-37) and (3-45), we can derive that

$$D = C_{OI}F_1 = C_{OI}C_{i}^{-1}D_i \quad (3-53)$$

Therefore, the Jacobian matrix $J$ which relates the input displacement to output displacement is derived as

$$J = C_{OI}C_{i}^{-1} \quad (3-54)$$

Because the double four-bar P joint has much higher stiffness in normal direction than in the working direction, $C_{BCD}^{E}(2,2) \gg C_{tran}^{E}(2,2)$, the element in $C_i$ (see Eq. (3-42) and (3-44)) is approximately written as

$$C_i(2,2) = C_i^{E}(2,2) = \frac{C_{vert}^{E}(2,2)(C_{BCD}^{E}(2,2) + C_{tran}^{E}(2,2))}{C_{vert}^{E}(2,2) + C_{BCD}^{E}(2,2) + C_{tran}^{E}(2,2)} \approx \frac{C_{vert}^{E}(2,2)C_{BCD}^{E}(2,2)}{C_{vert}^{E}(2,2) + C_{BCD}^{E}(2,2) + C_{tran}^{E}(2,2)} = \frac{d_{oy}}{f_{iy}} = C_{OI}(2,2) \quad (3-55)$$

Hence we obtain that

$$C_i \approx C_{OI} \quad (3-56)$$

So the Jacobian matrix is approximate to be a $2 \times 2$ identity matrix.

$$J = C_{OI}C_{i}^{-1} \approx I \quad (3-57)$$

### 3.4 WORKSPACE AND STRESS ANALYSIS

Let $D$ denotes the actual maximum steady-state output displacement of the PZT, then the workspace range of the stage is a $D \times D$ square if the stress due to bending moments and axial loads does not exceed the allowable stress of the material, and buckling phenomenon does not happen.
Unloaded PZT can only produce displacement. When used in a restraint, PZT can generate force. But PZT would be compressed when a force is applied to it, so force generation is always accompanied with a reduction in the output displacement. As shown in Fig. 3.9, the relationship between the actual and nominal maximum steady-state output displacement of the PZT can be expressed by

\[ D_{\text{nom}} - \frac{k_s D + f_{\text{pl}}}{k_p} = D \]  

(3-58)

where \( k_p \) is the stiffness of PZT, \( k_s \) is the stiffness of spring load on PZT, \( f_{\text{pl}} \) is preload on PZT and \( D_{\text{nom}} \) is nominal maximum displacement output without external restraint. \( k_s = K_f(1,1) \) when there is no external load from mobile platform.

Rearranging Eq. (3-58), we can obtain the expression for actual maximum steady-stage displacement output of PZT.

\[ D = \frac{k_p D_{\text{nom}} - f_{\text{pl}}}{k_p + k_s} \]  

(3-59)

The stress occurring in the flexure hinge should be analyzed to ensure that the hinges work properly. When a pure bending moment acts on a notch hinge around its rotation axis and generates a angular displacement \( \theta \), the maximum stress \( \sigma_{\text{max}} \) occurs at each outer surface of the thinnest part of the hinge, which can be calculated by [34]
\[
\sigma_{\text{max}} = \frac{E(1 + \beta)^{9/20}}{\beta^2 f(\beta)} \theta
\]  
(3-60)

where \( \beta = t/2r \) and

\[
f(\beta) = \frac{3 + 4\beta + 2\beta^2}{(1 + \beta)(2\beta + \beta^2)^2} + \frac{6(1 + \beta)}{(2\beta + \beta^2)^{5/2}} \tan^{-1} \sqrt{\frac{2 + \beta}{\beta}}
\]  
(3-61)

Let \( \theta_{\text{rt}}^m \) and \( \theta_{\text{rv}}^m \) represent the maximum angular displacements of the hinges in the transverse and vertical P joints respectively. Because \( \theta_{\text{rt}}^m \) and \( \theta_{\text{rv}}^m \) are extremely small, the relationship between them and the maximum output displacement \( D \) of the PZT can be approximately written as

\[
\theta_{\text{rt}}^m = \frac{D}{l_1}, \theta_{\text{rv}}^m = \frac{D}{l_2}
\]  
(3-62)

According to Eq. (3-60) and (3-62), the maximum stresses subject to the rotation of the hinges are derived as

\[
\sigma_{\text{rt}}^m = \frac{E(1 + \beta)^{9/20}D}{\beta^2 f(\beta)l_1}, \quad \sigma_{\text{rv}}^m = \frac{E(1 + \beta)^{9/20}D}{\beta^2 f(\beta)l_2}
\]  
(3-63)

To avoid the material failure, the stresses are limited as follows:

\[
S_f \sigma_{\text{rt}}^m \leq \sigma_y, S_f \sigma_{\text{rv}}^m \leq \sigma_y
\]  
(3-64)

where \( S_f \in (1, +\infty) \) is an assigned safety factor and \( \sigma_y \) denotes the yield strength of the material. Substituting Eq. (3-63) into (3-64), we can obtain the following constraint equations for the dimensions of the stage:

\[
\frac{E(1 + \beta)^{9/20}D}{\beta^2 f(\beta)l_1} \leq \frac{\sigma_y}{S_f}, \quad \frac{E(1 + \beta)^{9/20}D}{\beta^2 f(\beta)l_2} \leq \frac{\sigma_y}{S_f}
\]  
(3-65)

When the Y-direction PZT generates the maximum force, the maximum tensile stress subject to axial load will occur on the thinnest portions of flexure hinges 5, 6, 7 and 8 shown in Fig. 3.5, which can be calculated by

\[
\sigma_t^m = \frac{f_{iy1}}{S_{\text{min}}} = \frac{f_{iy}C_{\text{vert}}^E(2,2)}{S_{\text{min}}(C_{\text{vert}}^E(2,2) + C_{\text{BCD}}^E(2,2) + C_{\text{tran}}^E(2,2))}
\]  
(3-66)

\[
= \frac{K_f(2,2)Dc_{\text{vert}}^E(2,2)}{4wt(C_{\text{vert}}^E(2,2) + C_{\text{BCD}}^E(2,2) + C_{\text{tran}}^E(2,2))}
\]

where \( wt \) is multiplied by 4 because chains \( a, b, c \) and \( d \) bear the axial load at
the same time. The tensile stress should not exceed the allowable stress of the material, so the following equation is obtained.

\[ S_f \sigma_t^m \leq \sigma_y \] (3-67)

Substituting Eq. (3-66) into (3-67) generates another constraint equation for the dimensions of the stage:

\[ \frac{K_f(2,2)DC^E_{vert}(2,2)}{wt(C^E_{vert}(2,2) + C^E_{BCD}(2,2) + C^E_{tran}(2,2))} \leq \frac{4\sigma_y}{S_f} \] (3-68)

Besides the material failure subject to bending and axial load, buckling may occur in the flexure hinges when the axial compressive loads are large enough, and would lead to instability of the XY stage. Sometimes the flexure hinge buckles even when the maximum stress is lower than the yield stress of the material. The critical load \( P_{cr} \) that would cause buckling can be estimated by [12]

\[ P_{cr} = \frac{\pi^2EI_{min}}{(l_{cr}^2)} \] (3-69)

where \( I_{min} = wt^3/12 \) is the minimum moment of inertia for the flexure hinge, and \( l_{cr} = 2 \times 2r \) is the critical length of the hinge under fixed-free condition.

When the Y-direction PZT generates the maximum force, the maximum axial compressive load \( P_{c^m} \) occurs on flexure hinges 1, 2, 3 and 4 shown in Fig. 3.5, which can be calculated by

\[ P_{c^m} = \frac{K_f(2,2)DC^E_{vert}(2,2)}{4(C^E_{vert}(2,2) + C^E_{BCD}(2,2) + C^E_{tran}(2,2))} \] (3-70)

To avoid buckling of the flexure hinges, the maximum axial compressive load should stay smaller than the critical load.

\[ S_f P_{c^m} \leq P_{cr} \] (3-71)

Substituting Eq. (3-69) and (3-70) into (3-71), we derive another constraint equation for the dimensions of the stage:

\[ \frac{K_f(2,2)Dl_{cr}^2c^E_{vert}(2,2)}{I_{min}(C^E_{vert}(2,2) + C^E_{BCD}(2,2) + C^E_{tran}(2,2))} \leq \frac{4\pi^2E}{S_f} \] (3-72)
Dynamic analysis is carried out to discover the relationship between the motion of the XY stage and the actuation force. There are fundamentally two approaches for deriving dynamic equations of motion [35]: vector methods and energy methods. The former includes momentum principles, D’Alembert’s Principle and Kane’s Method. The latter uses theories such as Hamilton’s Canonical Equation, the Boltzmann-Hamel Equations, the Gibbs Equations and Lagrange’s Equations. Both of these two approaches lead to scalar equations, but vector methods develop the dynamic equation from vectors equations based on Newton’s laws of motion, and energy methods begin with scalar energy function. Lagrangian method, which depends on energy balance, is employed for the dynamics modeling of the micro-motion stage because it is easier to get the energy equations of compliant mechanism than vector equations.

![Figure 4.1: Limb A of the Stage](image)

The input-displacement variables $D_1 = [d_{ix} \ d_{iy}]^T$ are chosen as the generalized coordinates, so that the kinetic energy of the compliant mechanism can be expressed in terms of the generalized coordinates. Suppose that the kinetic energies are generated from the rigid links that connect the flexure hinges.

In the limb $A$ as shown in Fig. 4.1, the motion of link $g$ is translational, and the
others including links \(a, b, c, d, m, n, s\) and \(p\) undergo both translational and rotational motion. Hence the kinetic energy of limb \(A\) is expressed by

\[
T_A = \frac{1}{2} m_T \left(\frac{1}{2} \dot{d}_{ix}^2 + \frac{1}{2} \frac{m_T}{l_1^2} \frac{d_{ix}^2}{l_1^2} \right) + \frac{1}{2} m_T \left(\frac{1}{2} \dot{d}_{iy}^2 + \frac{1}{2} \frac{m_T}{l_1^2} \frac{d_{iy}^2}{l_1^2} \right)
\]

(4-1)

where

\[
m_T = m_a + m_b + m_c + m_d
\]

(4-2)

\[
m_V = m_m + m_n + m_s + m_p
\]

(4-3)

According to the double symmetric property, the kinetic energies of limbs \(B, C\) and \(D\) can be derived by

\[
T_B = \frac{1}{2} m_T \left(\frac{1}{2} \dot{d}_{ix}^2 + \frac{1}{2} \frac{m_T}{l_1^2} \frac{d_{ix}^2}{l_1^2} \right) + \frac{1}{2} m_T \left(\frac{1}{2} \dot{d}_{iy}^2 + \frac{1}{2} \frac{m_T}{l_1^2} \frac{d_{iy}^2}{l_1^2} \right)
\]

(4-4)

\[
T_C = T_A
\]

(4-5)

\[
T_D = T_B
\]

(4-6)

The mobile platform \(o\) move along \(x\) and \(y\) axes without rotation, so the kinetic energy of stage \(o\) is written as

\[
T_O = \frac{1}{2} m_o \left(\dot{d}_{ix}^2 + \dot{d}_{iy}^2 \right)
\]

(4-7)

Hence the kinetic energy of the whole mechanism is the sum of all the kinetic energy mentioned above, which is expressed as

\[
T = T_A + T_B + T_C + T_O = \left(\frac{4}{3} m_T + m_g + \frac{1}{3} m_V + \frac{1}{2} m_o \right) \dot{d}_{ix}^2
\]

\[
\left(\frac{4}{3} m_T + m_g + \frac{1}{3} m_V + \frac{1}{2} m_o \right) \dot{d}_{iy}^2
\]

(4-8)

The generalized force \(Q_j\) is the resultant force of input force and elastic force.

\[
Q_j = F_i - KD_i
\]

(4-9)

By substituting the kinetic and potential energy into Lagrange’s equation as follow

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{D}_i} \right) - \frac{\partial T}{\partial D_i} = Q_j = F_i - KD_i
\]

(4-10)
the dynamic equation of the motion can be derived as

\[ M\ddot{q} + KD_i = F_i \]  \hspace{1cm} (4-11)

where

\[ M = \text{diag}\left( \frac{8}{3}m_T + 2m_g + \frac{2}{3}m_v + m_o, \frac{8}{3}m_T + 2m_g + \frac{2}{3}m_v + m_o \right) \]  \hspace{1cm} (4-12)

is the equivalent mass matrix, and \( K = K_i \) is the equivalent stiffness matrix.

The dynamics equation of undamped free vibration of the compliant system can be derived as

\[ M\ddot{q} + KD_i = 0 \]  \hspace{1cm} (4-14)

Based on the theory of vibrations, the modal equation describing free vibration of the system can be written as

\[ (K - \lambda_i^2 I)\phi_i = 0 \]  \hspace{1cm} (4-15)

where \( \lambda_i^2 \) and \( \phi_i \) are the eigenvalue and eigenvector relevant to the \( i \)-th mode shape of the mechanism. Solving the characteristic equation

\[ |K - M\lambda_i^2| = 0 \]  \hspace{1cm} (4-16)

allows generation of the eigenvalue \( \lambda_i^2 \). The fundamental natural frequencies can be calculated by

\[ f_i = \frac{\lambda_i}{2\pi} = \frac{1}{2\pi}\sqrt{K(i,i)/M(i,i)} \]  \hspace{1cm} (4-17)
CHAPTER 5: OPTIMIZATION

According to above analysis, it is obvious that the dimensions are critical to the kinematic, static and dynamic performances of the micro-motion stage, such as stiffness, workspace, natural frequencies, etc. Hence the determination of the dimensions is an indispensable step to obtain an ideal structure. The factors constraining the dimensions include the performances of PZT, material properties, possibility of machining, permissible volume and so on. It is necessary to take all these factors into account.

![Figure 5.1: Dimensions of the Micro-motion Stage](image)

In this thesis, the dimensions are optimized to maximize the natural frequencies of the micro-motion stage with a specific thickness \( w \) because the natural frequencies are...
directly related to the dynamic performance. The dimensions to be optimized are \( r, t, l_1 \) and \( l_2 \) since they are the main dimensions that influence the natural frequency. The other dimensions including \( h_1, h_2, h_4, h_5, h_6 \) and \( h_7 \) as shown in Fig. 5.1 are set to be large enough in case of undesired deformation which would cause error to the micro-motion.

5.1 NUMERICAL SIMULATION

In order to detect the possible trends in the way the dimensions influence the natural frequencies, the analytical models formulated for compliance and natural frequencies calculation in Chapters 3 and 4 are utilized to carry out numerical simulation. The natural frequency changes as a function of the dimensions \( r, t, l_1 \) and \( l_2 \), when other dimensions are kept constantly. Three dimensional plots are generated as shown in Fig. 5.2 which clearly illustrates the variation tendencies of the natural frequency with respect to the mechanism dimensions. Each time when two parameters are arranged in a three-dimensional plot, the other two parameters take the constants as follows: \( r = 2 \text{ mm}, t = 0.5 \text{ mm}, l_1 = l_2 = 5 \text{ mm} \). It is indicated in Fig. 5.2 that the frequency increases nonlinearly with increasing \( t \) and decreasing \( r \), because...
in this situation the input stiffness is enhanced significantly while the mass slightly increases. Decreasing \( l_1 \) and \( l_2 \) leads to the increase of input stiffness and decrease of mass, and thus raises the frequency.

5.2 PARTICLE SWARM OPTIMIZATION AND RESULTS

Particle swarm optimization (PSO) is a computational technique developed by J. Kennedy and R. Eberhart [36]. It was inspired by social behavior of bird flocks and fish schools. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. PSO optimizes a problem by moving a number of particles around in the search-space according to simple mathematical formulae over the position and velocity of the particles. Each particle's movement is influenced by its local best known position, and is guided toward the best known positions in the search-space which are updated as better positions found by other particles. So the particle swarm is moved toward the best solutions.

PSO is initialized with a group of random particles. The particle \( i \) updates its velocity \( v_i \) and position \( x_i \) with the following equations:

\[
\begin{align*}
    v_i(k + 1) &= \phi(k)v_i(k) + c_1 \gamma_{1i}(p_i - x_i(k)) + c_2 \gamma_{2i}(G - x_i(k)) \\
    x_i(k + 1) &= x_i(k) + v_i(k + 1)
\end{align*}
\]  

(5-1)  

(5-2)

where \( \phi(k) \) is inertia function which determines the impact of previous velocities on the current velocities, \( c_1 \) and \( c_2 \) are local and global acceleration constants respectively, \( \gamma_{1i} \) and \( \gamma_{2i} \) are random numbers on the interval \([0,1]\) applied to \( i \)-th particle, \( p_i \) is the personal best position found by \( i \)-th particle and \( G \) is the global best position found by the swarm.

Compared with genetic algorithms (GA), PSO has advantages including no evolution operators such as crossover and mutation, ease of implement and few parameters to adjust [37]. Hence PSO is selected in this thesis for the optimization of dimensions.

The optimization problem can be summarized as follows:

1) Objective: maximize the natural frequency \( f \)
2) Parameters to be optimized: \( r, t, l_1 \) and \( l_2 \)

3) Constraints:
   a) parameters of flexure hinges: \( 0.05 \leq t/r \leq 0.65 \)
   b) input stiffness \( K_{i}(1,1) \leq 10\%k_p, K_{i}(2,2) \leq 10\%k_p \)
   c) constraint equations (3-65), (3-68) and (3-72) in Chapter 3
   d) ranges of parameters: \( 2 \text{ mm} \leq r \leq 6 \text{ mm} , \ 0.3 \text{ mm} \leq t \leq 2 \text{ mm} , \ 5 \text{ mm} \leq l_1 \leq 30 \text{ mm} \) and \( 5 \text{ mm} \leq l_2 \leq 30 \text{ mm} \)

The constraints are determined considering the factors as follows. The value of \( t/r \) is restrained to guarantee the accuracy of the compliance factors \( c_i \) chosen for Eq. (3-17). The input stiffness \( K_{i} \) is limited to no more than 10 percent of the PZT’s stiffness \( k_p = 50 \text{ N/μm} \) in order to keep the reduction of the output displacement small according to Eq. (3-58). To avoid plastic failures of the material, the constraint equations developed in Chapter 3 should also be satisfied with the safety factor \( S_f \) chosen as 2. The thinnest portion of the notch hinge is no less than 0.3 \text{ mm} because the micro-motion stage will be manufactured by WEDM technology, which cannot ensure a tolerance of \( \pm 0.01 \text{ mm} \) when the thickness is smaller than 0.3 \text{ mm} [10]. Besides, the upper bounds of the dimensions are set to achieve a compact structure.

The optimization process is carried out with a PSO MATLAB toolbox [38] combined with penalty function approach [39]. The population size of the particles is set to be 24 for this problem, the inertia weights have the values from inertial value 0.9 to final value 0.4 linearly in 1500 epochs. The local and global acceleration constants are assigned as \( c_1 = c_2 = 2 \). The following parameters are set for termination criterion: the maximum iteration value is 2000, the minimum global error gradient is \( 10^{-25} \), and the number of epochs before error gradient criterion terminates run is 250.

The searching for the optimal value terminates at the 1598 iterations because the global best value hasn’t change by at least \( 10^{-25} \) for 250 epochs, and the convergent process is illustrated in Fig. 5.3. The optimal results are: \( r = 2 \text{ mm}, t = 0.678 \text{ mm}, l_1 = 5 \text{ mm} \) and \( l_2 = 5 \text{ mm} \), which generate a micro-motion
stage with natural frequency $f = 730.27$ Hz. Considering the accuracy of WEDM, we modify the dimension of the thinnest part in the flexure hinge as $t = 0.67$ mm, and the micro-motion stage has a natural frequency of $719.49$ Hz and input stiffness of $4.85$ N/μm.

Figure 5.3: Convergent Process of PSO
In this chapter, finite element analysis (FEA) is carried out using ANSYS software to evaluate the stiffness, dynamic and decoupling performance of the micro-motion stage. The FEA result is compared with the result from compliance matrix method in order to verify the accuracy of analytical compliance equations. Some dimensions of the micro-motion stage are determined under the guidance of FEA in case of structural failure. The finite element model is built based on the designed geometrical structure. We adopt a two-dimension eight-node solid element (i.e. PLANE82) and the option of plane stress with thickness to mesh the model since the XY stage has a planar structure. To improve the computational accuracy, the smart mesh method is selected, and thus the nodes are non-uniformly distributed on the model and concentrated near the flexure hinges.

6.1 STATIC ANALYSIS OF THE PRISMATIC FLEXURE

As shown in Fig. 6.1, the finite element model of the double four-bar P joint is created according to the optimized dimensions. When a force $F$ along $x$ direction is applied to the motion stage, the P joint is deformed as shown in Fig. 6.2. The displacement of a certain node on the motion stage is considered as the displacement of the whole
motion stage. The relationship between the force and displacement, as well as the cross-axis error along \( y \) axis is plotted in Fig. 6.3. The FEA result is compared with the theoretical result generated by compliance matrix method, which is also presented in the figure. The result indicates a linear relationship between the force and displacement. The compliance of the double four-bar P joint in the working direction is \( 0.808 \, \mu m/N \), which is close to the theoretical value \( 0.818 \, \mu m \) with a difference about 1.2%. In addition, there is still a cross-error about 0.27% of the \( x \)-direction displacement, because the structure is not horizontally symmetric.

Figure 6.2: Deformation of a Double Four-bar P Joint under a Force

Figure 6.3: Comparison of the FEA Result with Theoretical Value
To compare the stiffness performance of double four-bar P joint with double parallelogram P joint, the finite element model of a double parallelogram P joint is established as shown in Fig. 6.4(a), which has the same links and flexure hinges as the double four-bar P joint. Fig. 6.4(b) shows the deformation due to a horizontal force exerted on the motion stage. In the force-displacement curve in Fig. 6.5, it is obvious that the stiffness of the double four-bar P joint is about four times as high as that of the double parallelogram P joint. The cross-axis error in the double parallelogram P joint is about 0.73%, which is higher than that of the double four-bar P joint. The reason is that all the flexure hinges are distributed in one side of the motion stage,
which causes unbalanced constraint forces.

6.2 STATIC ANALYSIS OF THE MICRO-MOTION STAGE

Nodes are constrained to have 
zero displacement in all directions

Figure 6.6: Finite Element Model of the Micro-motion Stage

Figure 6.7: Deformation of the Micro-motion Stage
Fig. 6.6 shows the finite element model of the whole stage. The four fixing holes at the four corners are constrained in all direction, and a force is applied to each driving point respectively. The flexure hinges are deformed and the mobile platform moves the way as Fig. 6.7 shows.

![Figure 6.8: Force-displacement Relationship of Y-direction Driving Point](image1)

![Figure 6.9: Force-displacement Relationship of X-direction Driving Point](image2)

When the stage is driven only in Y direction, the displacement of the Y-direction driving point increases linearly with the increasing driving force as shown in Fig. 6.8.
The input compliance calculated according to the FEA result is 0.225 μm/N, while the theoretical value obtained by compliance matrix method is 0.206 μm/N and the difference is 8.4%. Besides, the difference between $d_{iy}$ and $d_{oy}$ shows a displacement loss of 5.28% existed. The main reason for the displacement loss is that deformation exists in the links which are considered as rigid bodies in the compliance matrix method. The displacement loss increases as the actuation force increases, but it can be compensated via closed-loop control strategies. The displacement of the X-direction driving point is plotted against the driving force as shown in Fig. 6.9. The ratios of the translational displacements and rotational displacement at the X-direction driving point to the displacement output in Y direction are 0.85%, 1%, and 0.028 mrad/μm respectively, which indicates an ideal input decoupling property.

![Figure 6.10: Relationship between Input Force and Output Displacement](image)

To test the output decoupling performance, different driving forces of 0 N, 50 N and 100 N are exerted at the X-direction driving point. The relationship between the displacement of mobile platform and Y-direction driving force is shown in Fig. 6.10. The proportional relationship between the Y-direction displacement output $d_{oy}$ and Y-direction driving force $f_{iy}$ isn’t affected by the X-direction driving force $f_{ix}$, but
the value of $d_{oy}$ changes with $f_{ix}$ because the crosstalk caused by $f_{ix}$ reduces the displacement output in Y direction. Besides, the curves of the displacement output $d_{ox}$ against driving force $f_{iy}$ indicates that the crosstalk caused by $f_{iy}$ increases as $f_{iy}$ increases, but the increment is very small—about 0.02 μm per 10 N. A Y-direction displacement output about 21 μm would cause a crosstalk of 0.2 μm in X direction, so the cross-coupling is less than 1%, which indicates an excellent output decoupling property of the micro-motion stage.

The output compliance is tested by exerting an external force/moment on the mobile platform, and the force-displacement relationship is shown in Fig. 6.11. The output compliance in Y direction is 0.218 μm/N, and the rotational compliance around Z direction is 0.038 mrad/N·m. The corresponding theoretical value is 0.205 μm/N and 0.0397 mrad/N·m, respectively.

![Figure 6.11: Force-displacement Relationship of the Mobile Platform](image)

To test the stress that the compliant structure bear, an input force $F_t = [100 \text{ N} \ 100 \text{ N}]^T$ is applied to the micro-motion stage. The maximum von Mises stress is 48.2 MPa and occurs in the flexure hinge shown in Fig. 6.12. It is much lower than the yield stress of the material which is 502 MPa, so the micro-motion stage could work properly in a 21 μm × 21 μm workspace. When the
input force increases to \( F_I = [500 \ N \ 500 \ N]^T \), the maximum von Mises stress is 241 MPa, and the displacement output is 105.2 \( \mu \text{m} \). It indicates that the micro-motion stage has the potential to achieve a larger workspace of 105 \( \mu \text{m} \times 105 \mu \text{m} \) without material failure.

6.3 MODAL ANALYSIS

The modal analysis is carried out to test the dynamic performance of the micro-motion stage. The analysis generates several mode shapes and their corresponding frequencies, which reflect the dynamic response of the structure to the excitation. The first four mode shapes extracted by subspace method are shown in Fig. 6.13.

The translations of the micro-motion stage in Y and X directions occur in the first two modes. Because of the micro-motion stage’s symmetric structure, the first two modes have almost the same natural frequencies, which are 720.52 Hz and 721.02 Hz respectively. The corresponding theoretical values derived from Lagrange’s equation are both 719.42 Hz, and the difference between the FEA result and theoretical value is less than 0.25%. Rotation and twist occur in the third and forth modes, which have
high frequencies—1509 Hz and 5028.2 Hz. These two modes shapes are not considered in the analytical modeling because they are difficult to be modeled. There are also other mode shapes with higher frequencies, but only the first two modes are desirable since they have the lowest frequencies and thus are the most prominent modes which dominate all the other higher-frequency modes.

Figure 6.13: First Four Mode Shapes of the Micro-motion Stage
CHAPTER 7: CONTROL STRATEGY

The piezoelectric actuators have the advantages of high resolution, fast response, etc, but it also introduces nonlinearities into the micro-motion stage due to its inherent hysteresis and creep characteristics, which lead to undesirable inaccuracy or oscillations. Hence, in order to obtain a micro-motion stage of fine accuracy, the control strategy should handle the nonlinear effects including the hysteresis and creep existing in PZT, manufacturing tolerance and errors, etc. A variety of approaches have been proposed to cope with the nonlinearity of PZT. Typically, the hysteresis and creep are compensated by a model-based feed-forward compensator [40]. A commonly used control strategy which adopts this approach is integrating a feedback controller with a hysteresis compensator based on forward [41], [42] or inverse [43] Preisach model, which is a one of the most widely used hysteresis model that was first suggested by Ferenc Preisach in 1935. In addition, other methods such as robust control [44], neural network control [45], hysteresis observer-based control [46] and a self-tuning regulator combined with a feed-forward phase-lead compensator [47] also have been developed.

In this thesis, the control system is a PID feedback controller combined with a hysteresis compensator based on forward Preisach model because it has a simple structure and is easy to be implemented, and PID control also has wide applications in practice.

7.1 PREISACH MODEL OF Hysteresis

Hysteresis is a common phenomenon that occurs in magnetic materials, ferromagnetic materials and ferroelectric materials. In piezoelectric actuators, the hysteresis causes a lag in the values of resulting displacement output when the PZT is applied a varying electric field. Fig. 7.1 shows a typical hysteresis loop of a PZT. When the input voltage is changing, the change of displacement lags behind the change of voltage.
Hence the curve with a decreasing voltage is not the same as the one with an increasing voltage.

![Figure 7.1: A Typical Hysteresis Loop of a PZT](image)

The classic Preisach model that describes the hysteresis behavior of a PZT can be expressed in the following mathematical form:

\[
y(t) = \int_{\alpha \geq \beta} w(\alpha, \beta) \gamma_{\alpha\beta}[u(t)] \, d\alpha d\beta
\]  

(7-1)

where \(y(t)\) is the output displacement of a PZT, \(\gamma_{\alpha\beta}[u(t)]\) is the hysteresis relay, \(u(t)\) is the input voltage, and \(w(\alpha, \beta)\) is a weighting function in the Preisach model which describes the relative contribution of each relay to the overall hysteresis. The hysteresis relay can be expressed by a rectangular loop on the input-output diagram shown in Fig. 7.2. Each relay has a pair of switching values \((\alpha, \beta)\) with \(\alpha \geq \beta\),
which correspond to the “up” and “down” switching values of input, respectively. The output of the relay $\gamma_{\alpha\beta}[u(t)]$ has only two values, 0 and 1 [48]. The value is determined by the input voltage as follow:

$$
\gamma_{\alpha\beta}[u(t)] = \begin{cases} 
1 & u \geq \beta \\
0 & u \leq \alpha \\
k & \alpha < u < \beta 
\end{cases}
$$

(7-2)

where $k = 0$ if the last time $u$ was outside of the boundaries $\alpha < u < \beta$, it was in the region of $u \leq \alpha$; and $k = 1$ if the last time $u$ was outside of the boundaries $\alpha < u < \beta$, it was in the region of $u \geq \alpha$.

![Figure 7.3: Interpretation of Preisach Model](image)

The Preisach model can be considered as a superposition of many parallely connected hysteresis relays with given weights as illustrated in Fig. 7.3. As the input voltage $u$ varies with time $t$, each individual relay $\gamma_{\alpha\beta}$ adjust its output—either 0 or 1—according to the input value, and the weighted sum of all the relay’s outputs generates the overall displacement output $y$.

There is a one-to-one relationship between the hysteresis relays and switching-value pairs $(\alpha, \beta)$. So the Preisach model can be geometrically interpreted by a $\alpha$-$\beta$ plane as shown in Fig. 7.4. Because $\alpha \geq \beta$, all the hysteresis relays are located in a half of the plane. $\alpha_0$ and $\beta_0$ represent the upper and lower limits of the input voltage, respectively. Each relay is mapped to a point in the plane, whose coordinate corresponds to the switching-value pair of the relay.
Figure 7.4: A $\alpha\beta$ Plane of the Preisach Model

Figure 7.5: Increasing Input Voltage

Figure 7.6: Decreasing Input Voltage
As the input voltage \( u(t) \) increases from 0 to \( \alpha_1 \), all the relays \( \gamma_{\alpha \beta} \) with switching value \( \alpha \) lower than \( \alpha_1 \) are switched from 0 to 1, so the plane is divided into two parts as shown in Fig. 7.5. The outputs of the relays in region \( P^+ \) are all 1, and the outputs of the relays in region \( P^- \) are all 0. The overall displacement output \( y^{\alpha_1} \) is the weighted sum of the relays in region \( P^+ \).

\[
y^{\alpha_1} = \int_{P^+} w(\alpha, \beta) \gamma_{\alpha \beta}(u(t)) d\alpha d\beta + \int_{P^-} w(\alpha, \beta) \gamma_{\alpha \beta}(u(t)) d\alpha d\beta
\]

(7-3)

More and more relays are switched to 1 as the input voltage increases, so the horizontal boundary of region \( P^+ \) moves up and the overall output \( y(t) \) increases.

As the input voltage \( u(t) \) decreases from \( \alpha_1 \) to \( \beta_1 \), the relays \( \gamma_{\alpha \beta} \) with switching value \( \beta \) higher than \( \beta_1 \) are switched from 1 to 0, so part of region \( P^+ \) is converted to \( P^- \). More and more relays are switched to 0 as the input decreases, so the vertical boundary of region \( P^+ \) moves left and the output \( y(t) \) decreases (see Fig. 7.6).

![Figure 7.7: Numerical Implementation of Preisach Model for Increasing Input](image)

However, the continuous Preisach model is difficult to be implemented because it is
complicated to obtain the weighting function directly. To avoid this problem, the numerical implement of Preisach model is induced. Fig. 7.7 shows a hysteresis curve with increasing input and its corresponding $\alpha$-$\beta$ plane. The curve has a series of input maxima $\alpha_k$ ($k = 1, 2, \cdots, n$) and minima $\beta_k$ ($k = 0, 1, \cdots, n$). The output displacement $y(T)$ is the weighted sum of the relays in region $P^+$ which can be divided into several trapezoids and a triangle. The output in each trapezoid $Q_k$ can be expressed as a difference of the outputs in two triangular areas:

$$y_{Q_k} = \int_{\beta_{k-1}}^{\beta_k} \int_{\alpha}^{\alpha_k} w(\alpha, \beta) \, d\alpha \, d\beta$$

$$= \int_{\beta_{k-1}}^{\alpha_k} \int_{\beta}^{\alpha} w(\alpha, \beta) \, d\alpha \, d\beta - \int_{\alpha}^{\alpha_k} \int_{\beta}^{\beta_k} w(\alpha, \beta) \, d\alpha \, d\beta$$

$$= Y(\alpha_k, \beta_{k-1}) - Y(\alpha_k, \beta_k)$$

where the Preisach function $Y(\alpha_k, \beta_k)$ denotes the output of all the relays in the triangular region consisting of three vertices $(\beta_k, \beta_k)$, $(\alpha_k, \beta_k)$ and $(\alpha_k, \alpha_k)$.

Hence the output $y(T)$ is derived by

$$y(T) = \sum_{k=1}^{n} \left[ \int_{\beta_{k-1}}^{\beta_k} \int_{\alpha_k}^{\alpha} w(\alpha, \beta) \, d\alpha \, d\beta + \int_{\beta}^{u(T)} \int_{\beta_k}^{\beta} w(\alpha, \beta) \, d\alpha \, d\beta \right]$$

$$+ \sum_{k=1}^{n} \left[ Y(\alpha_k, \beta_{k-1}) - Y(\alpha_k, \beta_k) \right] + Y(u(T), \beta_n)$$

(7-5)

Figure 7.8: Numerical Implementation of Preisach Model for Decreasing Input
In the same way, for the curve as shown in Fig. 7.8, which has a decreasing input, a series of input maxima \( \alpha_k \) \((k = 1, 2, \cdots, n)\) and minima \( \beta_k \) \((k = 0, 1, \cdots, n - 1)\), the output can be derived by

\[
y(T) = \sum_{k=1}^{n-1} \int_{\beta_{k-1}}^{\beta_k} \int_{\alpha_{k-1}}^{\alpha_k} w(\alpha, \beta) \, d\alpha \, d\beta \\
+ \left( \int_{\beta_{n-1}}^{\alpha_n} \int_{\alpha_{n-1}}^{\alpha_n} w(\alpha, \beta) \, d\alpha \, d\beta - \int_{u(T)}^{\alpha_n} \int_{\beta_{n-1}}^{\beta_n} w(\alpha, \beta) \, d\alpha \, d\beta \right) \\
= \sum_{k=1}^{n-1} (Y(\alpha_k, \beta_{k-1}) - Y(\alpha_k, \beta_k)) + Y(\alpha_n, \beta_{n-1}) - Y(\alpha_n, u(T)) \tag{7-6}
\]

Figure 7.9: Data Mesh Determined by Experiments

Since the output \( y(T) \) can be expressed as a function of \( Y(\alpha_k, \beta_k) \), it is critical to determine \( Y(\alpha_k, \beta_k) \) for each point in the \( \alpha-\beta \) plane. In this thesis, a data mesh is created for a finite number of grid points in the \( \alpha-\beta \) plane as shown in Fig. 7.9(a), each point has its corresponding value \( Y(\alpha_k, \beta_k) \), and the value of \( Y(\alpha_k, \beta_k) \) is derived by

\[
Y(\alpha_k, \beta_k) = y^{\alpha_k} - y^{\alpha_k \beta_k} \tag{7-7}
\]

where \( y^{\alpha_k} \) and \( y^{\alpha_k \beta_k} \) are experimentally measured according to Fig. 7.9(b). Then the output \( y(T) \) is calculated by Eq. (7-5) and (7-6).

However, some actual values of input voltage are not included in the grid points, which means that there are no corresponding experimentally measured values. The
actual point would be located in a square or triangular cell as shown in Fig. 7.10. To address this problem, the following interpolation functions are used to compute the values of $Y(\alpha_k, \beta_k)$ for the points located in square or triangular cells [49]:

$$Y(\alpha_k, \beta_k) = \begin{cases} c_0 + c_1 \alpha_k + c_2 \beta_k + c_3 \alpha_k \beta_k & \text{for square cells} \\ c_4 + c_5 \alpha_k + c_6 \beta_k & \text{for triangular cells} \end{cases}$$ (7-8)

where the coefficients $c_i$ are calculated by matching the output values at the vertices of the square or triangular cell.

Figure 7.10: Actual Points Located in Square or Triangular Cells

7.2 CONTROLLER DESIGN

Although the output displacement $y(t)$ of the PZT caused by an input voltage $u(t)$ can be predicted by employing the numerical Preisach model, only a feed-forward compensator based on the hysteresis model is not enough to achieve robust and accurate tracking control, because the accuracy of the Preisach model utterly depends on the experimental data. Therefore, a PID controller is integrated with the Preisach model-based feed-forward compensator. Moreover, a single-input-single-output (SISO) control strategy can be applied to each moving direction of the micro-motion stage independently, since the stage has an excellent decoupling property. Fig. 7.11 shows the block diagram of the control system.

In the feed-forward loop, the control voltage $u_d(t)$ is predicted by the compensator
according to the desired displacement $y_d(t)$. In the feedback loop, the desired displacement is compared with the measured output $y(t)$ of the micro-motion stage, and then the error signal $e(t)$ is sent to the PID controller, which generates an additional voltage $u_{PID}(t)$. Finally, the sum of the predicted voltage $u_d(t)$ and the additional voltage $u_{PID}(t)$ is sent to the PZT. Thus the input voltage $u(t)$ of the PZT can be written as

$$u(t) = u_{PID}(t) + u_d(t) = \left\{ k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{d}{dt} e(t) \right\} + u_d(t) \quad (7-9)$$

where $k_p$, $k_i$ and $k_d$ are the proportional, integral and derivative gains, respectively.

Because the controller is implemented in discrete time by a computer, the controller is discretized as:

$$u(kT) = u_{PID}(kT) + u_d(kT)$$

$$= k_p e(kT) + k_i \sum_{j=0}^{k} e(kT)T + k_d \left( e(kT) - e((k - 1)T) \right) + u_d(kT) \quad (7-10)$$

where $T$ is the sampling period and $k = 1, 2, \cdots$.

![Figure 7.11: Block Diagram of the Control System](image)

Fig. 7.12(a) shows the algorithm of the compensator. In the beginning, a set of desired output displacements $y_d(k)(k = 1, 2, \cdots, N)$ is specified. The corresponding set of reference input voltages $u_r(k)$ is calculated according to the nominal linear displacement-voltage relationship of the PZT, which is illustrated in Fig. 7.12(b). Then the voltages $u_r(k)$ are used to generate a set of reference displacement $y_r(k)$ via Preisach model. Each desired displacement $y_d(k)$ is compared with the reference displacement $y_r(m)$, and the index number of the reference displacement, $m$, is
increased or decreased according to the trend of the difference between $y_d(k)$ and $y_r(m)$, until the nearest $y_r(m)$ to $y_d(k)$ is found. Then the corresponding $u_r(m)$ is taken as the desired voltage $u_d(k)$. In this way, a set of desired voltages is predicted for the set of desired displacement. All the calculation in the feed-forward compensator is finished before sampling action starts, in order to minimize the time consumed on the calculation of control voltage $u(t)$ within each sampling interval.

Figure 7.12: Algorithm of the Compensator
CHAPTER 8: PROTOTYPE FABRICATION AND EXPERIMENTS

8.1 PROTOTYPE FABRICATION AND EXPERIMENTAL SETUP

A prototype of the designed micro-motion stage is manufactured by wire electrical discharge machining (WEDM) technique with aluminum alloy AL7075-T6, and the machining precision is ±0.015 mm. The main parameters are shown in Table 8.1.

Table 8.1: Main Parameters of the Prototype

<table>
<thead>
<tr>
<th>Dimensions (mm)</th>
<th>r</th>
<th>t</th>
<th>h</th>
<th>w</th>
<th>l₁</th>
<th>l₂</th>
<th>h₁</th>
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<td>4.67</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>10</td>
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</table>

<table>
<thead>
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<th>h₃</th>
<th>h₄</th>
<th>h₅</th>
<th>h₆</th>
<th>h₇</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

Material parameters

<table>
<thead>
<tr>
<th>Young’s modulus</th>
<th>Yield strength</th>
<th>Poisson’s ratio</th>
<th>Density ρ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (GPa)</td>
<td>$σ_y$ (MPa)</td>
<td>$ν$</td>
<td>$2.81 \times 10^3$</td>
</tr>
<tr>
<td>71.7</td>
<td>503</td>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8.1 shows the whole system consisting of the mechanical and control modules, and the hardware is connected in the way shown in Fig. 8.2. The micro-motion stage is fixed on a vibration-isolation table in order to dampen the vibration from environment. Two identical PZTs and the corresponding controllers (BPC002) from Thorlabs Inc. are chosen to drive the stage. The specifications of the PZT are listed in Table 8.2. Two laser sensors (Microtrak II) and the controllers from MTI Instruments Inc. are used to measure the output displacements at two blocks mounted on the mobile platform. The laser head (LTC-025-03) has a working range of ±1 mm with a resolution of ±0.12 μm. A data acquisition board (DAQ NI PCI-6289), which has thirty two analog inputs and four 16-bit analog outputs, is installed in the computer.
It obtains voltage signal from the laser controller or output voltage signal to the PZT controller through an external connector (BNC-2120). The control program is written and run to realize the control strategy with MATLAB software in the computer.

Figure 8.1: The Whole System of the Micro-motion Stage
Figure 8.2: Block Diagram of Hardware Connection

Table 8.2: Specifications of the PZTs

<table>
<thead>
<tr>
<th>Item No.</th>
<th>PAS020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm) × Length (mm)</td>
<td>12.7 × 42</td>
</tr>
<tr>
<td>Travel (µm)</td>
<td>20.0</td>
</tr>
<tr>
<td>Resolution (nm)</td>
<td>20.0</td>
</tr>
<tr>
<td>Input Voltage (VDC)</td>
<td>0 – 75</td>
</tr>
<tr>
<td>Blocking force</td>
<td>1000 N at 60 V, 1500 N at 75 V</td>
</tr>
<tr>
<td>Stiffness(N/µm)</td>
<td>50</td>
</tr>
</tbody>
</table>

8.2 OPEN-LOOP TEST

Figure 8.3: Input Voltage to Each PZT
Figure 8.4: Open-loop Voltage-displacement Relationship and Crosstalk

Under open-loop condition, each time the micro-motion stage is driven by one PZT with a 0.05 Hz sinusoidal input voltage as shown in Fig. 8.3, the displacements in both directions are measured to determine the motion range and crosstalk in each direction. Fig. 8.4 clearly shows the hysteresis property of the PZT. The micro-motion stage has a motion range of 19.2 μm in X direction and 18.8 μm in Y direction. The range of crosstalk is [−0.5 μm, 0.5 μm] and [−0.6 μm, 0.2 μm] in X and Y direction respectively. Hence the micro-motion stage has a workspace of 19.2 μm × 18.8 μm with crosstalk less than 5%. The hysteresis properties of the two PZTs are different, which leads to different motion ranges in each direction. It also indicates that a Preisach model should be built for each PZT respectively. The maximum hysteresis difference is about 15% of the motion range. The PZT cannot reach its full stroke because the controller can only output 71.7 V when the DAQ board reaches its maximum output voltage 10 V.
A step signal of 9.6 μm is sent to the X direction under open-loop condition in order to test its dynamic response characteristics. As shown in Fig. 8.5, the step response of the system in X direction has an overshoot of 9.88% and the settle time is about 0.35 s. The value stabilize at 11.2 μm, which is much higher than the desired value due to the nonlinear property of the PZT. Fig. 8.6 shows the open-loop step response
in Y direction with an input signal of 9.4 μm, which has a overshoot of 15.4%. The value stabilizes at 11.5 μm in 0.35 s. The open-loop step test indicates that the system without closed-loop control stabilizes fast but has a high overshoot, and the output error is high. It is necessary to employ an effective closed-loop controller.

8.3 HYSTERESIS IDENTIFICATION

To obtain the numerical Preisach model of hysteresis, a sinusoidal voltage signal with decreasing amplitude (see Fig. 8.7) is input to the PZT. A hysteresis curve with 10 loops for each direction is generated as shown in Fig. 8.8. Each period of the input sinusoidal signal corresponds to one output hysteresis loop. A number of points are selected from the loops at a fixed step. The data mesh (see Fig. 8.9) of the Preisach function $Y(\alpha, \beta)$ in $\alpha$-$\beta$ plane is derived according to the selected data points. Then the hysteresis curve is rebuilt to verify the accuracy of the model. Fig. 8.10 and Fig. 8.11 show the comparison of the hysteresis curves obtained from experiments and from Preisach models. The similarity of them indicates that the Preisach model has enough accuracy to predict the hysteresis.

![Figure 8.7: Input Voltage for Preisach Model Identification](image-url)
Figure 8.8: Hysteresis Curves with 10 Loops

Figure 8.9: Data Meshes in $\alpha$-$\beta$ Plane
Figure 8.10: Comparison of the Experimental and Preisach Models in X direction

Figure 8.11: Comparison of the Experimental and Preisach Models in Y direction
After obtaining the Preisach model, the feed-forward compensator is set up and integrated with a PID controller to form a closed-loop control system. Fig. 8.12 and Fig. 8.13 show the step response of the closed-loop system in X and Y direction, respectively. Compared with open-loop system, the closed-loop response stabilizes in a much shorter time—less than 0.125 s, and the overshoot is eliminated. Moreover,
the error between the desired and actual output value is small—less than 2.2%.

Figure 8.14: Point Positioning of the Micro-motion Stage

The positioning accuracy of the micro-motion stage is tested by increasing the inputs in each direction gradually to a certain value (9.4 μm, 9.4 μm). Then the position of the mobile platform is recorded. The experiment is repeated fifty times and the result is shown in Fig. 8.14. All the points are located in a 0.35 μm × 0.4 μm rectangle surrounding the target point, which shows excellent positioning performance of the micro-motion stage.

Figure 8.15: Sinusoidal Motion Tracking with Open-loop Control
Figure 8.16: Sinusoidal Motion Tracking with Feed-forward Control

Figure 8.17: Sinusoidal Motion Tracking with PID Feedback Control

To test the effectiveness of the controller, a 0.04 Hz sinusoidal signal is input to X direction of the stage when the controller is one of the following types: an open-loop controller, a feed-forward compensator, a PID feed-back controller or a PID feed-back controller integrated with a feed-forward compensator. The tracking results are shown in Fig. 8.15—Fig. 8.18, respectively. The open-loop control has the worst
accuracy—the error can reach 29.5%, since the hysteresis exists and no feedback is used to correct the output displacement. When only with a feed-forward compensator, the hysteresis is reduced remarkably—the error is below 7.1%. But error is still conspicuous because the hysteresis model is built utterly depending on the experimental data, which results in the compensator’s weak robustness. The tracking result with a PID feedback controller shows a better accuracy—the error is below 5.9%. However, the hysteresis still exists, which can be seen clearly from the relationship between the input and output displacement. The error could be even larger when the velocity rises. By integrating the PID controller with the compensator, the tracking result shows a highest accuracy—the error is below 2.95%. The relationship between the input and output displacement almost becomes linear.

![Figure 8.18: Sinusoidal Motion Tracking with PID Feedback Controller Integrated with Feed-forward Compensator](image)

Fig. 8.19 shows that the actual control voltage signal sent to the PZT is composed of two parts: the voltage signals from the compensator and from the PID feedback controller. The compensator outputs the predicted voltage, and the PID controller is responsible for generating a relatively lower correcting voltage according to the error between the target displacement and actual output.
Contouring is implemented by driving the micro-motion stage along a circular trajectory with a diameter of 16.9 μm. The two PZTs are driven by two sinusoidal signals with same amplitudes but different phases. The tangential velocities are set to 8.8 μm/s, 4.4 μm/s, 2.2 μm/s and 1.1 μm/s respectively by choosing different
discrete steps for the input signal. The experimental result is shown in Fig. 8.20—Fig. 8.23, which indicates that no significant contouring distortion occurs during the motion. As the velocity decreases, the tracking error of the sinusoidal signal is reduced so that the contouring error decreases.

Figure 8.21: Circular Contouring with a Velocity of 4.4 µm/s

Figure 8.22: Circular Contouring with a Velocity of 2.2 µm/s
Figure 8.23: Circular Contouring with a Velocity of 1.1 μm/s
CHAPTER 9: CONCLUSIONS AND FUTURE WORKS

9.1 SUMMARIES

Micro-motion XY stage is a common tool used to complement positioning and manipulation with ultra-high precision in micro- and nanotechnology. However, most of the existing stages are serial structures, or parallel structures with input or/and output coupling. With the purpose to design a completely decoupled micro-motion stage, a compliant parallel 4-PP mechanism is proposed. Flexure hinges are adopted instead of conventional mechanical joints and PZT is employed to drive the stage.

The kinetostatic analysis is started from the compliance modeling of the compliant parallel mechanism. By considering the flexure hinge as having three DOFs, i.e. two translational and one rotational freedom, the output and input compliance of the stage is derived via simplified compliance matrix method. The kinematic analysis shows that the Jacobian matrix which relates the input displacement to output displacement is approximate to an identity matrix. The workspace of the micro-motion stage depends on the actual output of the PZT if the stress generated in the flexure hinges does not exceed the material's yield strength and buckling phenomenon does not happen. Three dimensional constraints are derived to avoid material failure.

On the premise that the stage has an ideal decoupling property, the dynamic model is built with Lagrange’s equation. The input-displacement variables are chosen as the generalized coordinates. It turns out that the compliant XY stage can be taken as a second-order linear system.

The dimensions are optimized in order to maximize the natural frequencies of the stage. Besides the constraints derived from stress analysis, the possibility of machining and permissible volume are also the factors constraining the dimensions. The optimization is carried out using PSO algorithm combined with penalty function
approach, which generates a XY stage with a theoretical natural frequency of 719.49 Hz and input stiffness of 4.85 N/μm.

To evaluate the static and dynamic performance of the stage with optimal dimensions, FEA is carried out using ANSYS software. The FEA result of the two types of P joints, i.e. double parallelogram P joint and double four-bar P joint, indicates that the double four-bar structure has better stiffness performance and a smaller cross-axis error than the double parallelogram structure. The FEA result of the entire mechanism shows that the XY stage is well decoupled and has an input compliance of 0.225 μm/N. The stage has the potential to achieve a 105 μm × 105 μm square workspace. The first vibrational mode is a translational deformation in Y direction, and the frequency is about 720.52 Hz.

Hysteresis is a nonlinear property commonly existed in the PZT. Traditional PID feedback controller cannot reduce the hysteresis to an acceptable level, especially in high-speed control. Hence, a feed-forward compensator based on Preisach model is integrated with a PID feedback controller to construct the control system for the micro-motion stage.

Finally, a prototype of the completely decoupled XY stage is fabricated with aluminum alloy AL7075-T6 using WEDM technique. Two identical PZTs with the maximum output of 20 μm are selected as the actuators, and two laser sensors are used to measure the displacement output of the stage. The control strategy is implemented by a PC with MATLAB software. Open-loop test shows the actual workspace of the XY stage is 19.2 μm × 18.8 μm, and coupling between the two axes is less than 5%. The numerical Preisach model for each PZT is built according to the data mesh obtained from open-loop experiments. The hysteresis curve rebuilt by the Preisach model matches the experimental curve with a high similarity, which indicates a well accuracy of the model. With the closed-loop controller, the positioning test is carried out and the result shows that the actual position is located in a 0.35 μm × 0.4 μm rectangle surrounding the target position. The micro-motion
system has a tracking error below 2.95% for a 0.04 Hz sinusoidal input. By comparing the sinusoidal tracking results from different controllers, the effectiveness of the feed-forward PID controller is verified. When a circular trajectory with a diameter of 16.9 μm is tracked in various velocities of 1.1 μm/s, 2.2 μm/s, 4.4 μm/s and 8.8 μm/s, the micro-motion stage shows an increasing contouring error as the velocity increases.

9.2 FUTURE WORKS

Resolution, workspace and velocity are three critical parameters of a XY micro-motion stage. Unfortunately, these three parameters are often in conflict with each other. For example, the workspace could be enlarged with a displacement amplifier such as a lever [50] or a bridge-type amplifier [51], and the velocity would be raised as well, but this would lead to the drop of resolution and accuracy, and total volume probably would increases. High velocity requires a high natural frequency, which causes a high stiffness and thus constrains the workspace. In view of these conflicts, the future works of this research topic is introduced as follows:

The workspace can be enlarged with a PZT of a higher displacement output, or with a displacement amplifier. Compared with the latter, the former method can avoid the loss of resolution, but the economical cost is high. In addition, a displacement amplifier would lead to the increase of input stiffness, and thus improve the requirement for a PZT with high stiffness. Similarly, the accuracy of the micro-motion stage can be improved by using actuators and sensors with higher resolution, e.g. capacitive sensors, but this would lead to a high economical cost. Hence the decision would be made according to the practical applications.

The right-circular flexure hinges can be replaced by beam flexures, which could lead to a larger workspace and lower input stiffness. The natural frequency probably would be reduced but depends on the actual dimensions. Further research could be conducted on it.
The velocity of the micro-motion stage in this thesis is restricted by the implementation environment of the control system, in which the processing of each sample point takes about 0.125 s. To improve the velocity without the drop of accuracy, an x-PC target environment or a dSPACE controller could be adopted.

The actual hysteresis in the PZT is rate-dependent, which means the hysteresis changes as the frequency of the input voltage signal changes. The derived Preisach model is no longer accurate when the input frequency changes largely, so the hysteresis should be remeasured to generate a new model. To address this problem, a rate-independent control strategy can be employed, such as dynamic Preisach model, neural networks, hysteresis observer and robust controller.


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