Numerical Methods for Pricing Callable Bonds

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

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Supervisor

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Abstract

A callable bond is a bond that allows the issuer to buy back the bonds from the bond holders at pre-specified prices on the pre-specified call dates. Therefore, a callable bond is a straight bond embedded with a call of European option (a single call date) or Bermudan option (several call dates). However, this option is an integral part of a bond, and cannot be traded alone, and hence, its prices cannot be observed. Thus, the callable bond pricing must be involved in the pricing of the corresponding option.

There are three different approaches for pricing callable bonds. The first approach (thereafter the I approach) is based on the Black-Derman-Toy model, which was presented in [3] (1990), with the discrete simulation of binary tree. The second approach (thereafter the II approach) is to obtain a partial differential equation (PDE) subject to appropriate boundary conditions based on the equilibrium interest rate model. Since it is very difficult to analytically solve this PDE, some different discretizations and different numerical methods have been proposed.

Meanwhile, in the last decade, many new numerical schemes for simulations of interest rate models, especially, the Cox-Ingersoll-Ross (CIR) interest rate model, have been proposed. In this paper, based on these new simulation techniques, we present a new Monte Carlo method as the third approach (thereafter the III approach) to numerically price the European-type and Bermudan-type callable bonds with notice dates.

This approach is the original contribution of this thesis and has many advantages. For instance, via simulation techniques, pricing interest rate derivatives can be more flexible and precise due to the variable simulation techniques developed rapidly for advanced continuous interest rates models,
and more efficient due to the potential parallel computation for large scale simulation.

In addition, both the callable bond price and the “break-even interest rate” for issuer’s decision on every notice dates can be valued. All of the theoretical deductions are presented completely with detailed elaborations of conditional expectations and corresponding Monte Carlo simulations. And the corresponding algorithms are given completely for callable bonds pricing as well. By introduction of tree idea for debt flow from data structure science, the algorithms become concise and clear. Furthermore, the formula for pricing callable bonds numerically is given explicitly, thus let the new method can be put into practice.

Later, the numerical experiments show that this method works very well for callable bonds under the CIR interest rate model. With comparisons of the performance of the four new simulation techniques, BMM method and DC schemes are more efficient than others. The numerical results also demonstrate that options embedded in callable bonds are non-negative.

To our knowledge, no previous scientific or technical literatures present this approach to price interest rates derivatives, especially callable bonds.
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DECLARATION

The author declares that this thesis represents his own work with Professor Deng Ding, the author’s supervisor. All the work is done under the supervision of Professor Ding during the period 2009-2011 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author’s knowledge, by anyone else.

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