GENERAL DOWEL EQUATIONS FOR CALCULATING LATERAL CONNECTION VALUES

TECHNICAL REPORT 12
(ISSUES RELATED TO THE 1997 NDS®)

American Forest & Paper Association
The North American wood products industry is dedicated to building a sustainable future

Only one primary building material comes from a renewable resource; cleans the air and water, providing habitat, scenic beauty and recreation as it grows; utilizes nearly 100% of its resource for products; is the lowest of all in energy requirements for its manufacturing; creates fewer air and water emissions than any of its alternatives; and is totally reusable, recyclable and 100% biodegradable: wood. And it has been increasing in US net reserves since 1952, with growth exceeding harvest in the US by more than 30%.

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Notation

- **D**: dowel shank diameter, in.
- **D_s**: dowel diameter at max. stress in side member, in.
- **D_m**: dowel diameter at max. stress in main member, in.
- **D_t**: fastener root diameter, in.
- **F_b**: dowel bending strength, psi
- **F_b,pl**: proportional limit dowel bending strength, psi
- **F_b,5%**: 5% offset dowel bending strength, psi
- **F_b,ult**: ultimate dowel bending strength, psi
- **F_c**: dowel bearing strength, psi
- **F_c,pl**: proportional limit dowel bearing strength, psi
- **F_c,5%**: 5% offset dowel bearing strength, psi
- **F_c,ult**: ultimate dowel bearing strength, psi
- **F_e**: dowel bearing strength parallel to grain, psi
- **F_e,c**: dowel bearing strength perpendicular to grain, psi
- **F_e,s**: side member dowel bearing strength, psi
- **F_e,m**: main member dowel bearing strength, psi
- **F_u**: tensile strength, psi
- **F_y**: tensile yield strength, psi
- **G**: specific gravity
- **K_D**: diameter coefficient for wood screw, nail and spike connections
- **K_θ**: angle to grain coefficient for bolt and lag screw connection
- **M_s**: side member dowel moment resistance, in.-lbf.
- **M_m**: main member dowel moment resistance, in.-lbf.
- **P**: nominal lateral connection value, lbs.
- **P_c,5%**: nominal 5% offset lateral connection value, lbs.
- **Z,Z’**: nominal and allowable lateral design value for a single fastener connection, lbs.
- **f'_c**: concrete compressive strength, psi
- **g**: gap between members, in.
- **l_s**: side member dowel bearing length, in.
- **l_m**: main member dowel bearing length, in.
- **q_s**: side member dowel-bearing resistance, lbs./in.
- **q_m**: main member dowel-bearing resistance, lbs./in.
- **t**: thickness, in.
PART I. GENERAL DOWEL EQUATIONS

1.1 Introduction

The yield limit equations specified in the National Design Specification® (NDS®) for Wood Construction (AF&PA, 1997) for bolt, lag screw, wood screw, nail, spike and drift pin connections provide a mechanics-based approach for connection design. This approach, which was incorporated in the NDS for Wood Construction in 1991, permits the designer to determine effects of member thickness, member strength, fastener size, and fastener strength on lateral connection values for the majority of connections found in wood construction. This report covers calculation of lateral values for single dowel type fastener connections using a generalized and expanded form of the NDS yield limit equations. These general dowel equations apply to NDS connection conditions, but also permit rational and consistent treatment of gaps and fastener moment resistance, and consideration of various connection limit states. General information is provided in Part I of this report. Part II contains example problems and Part III provides equation derivations.

1.2 Calculation of Lateral Connection Values

The general dowel equations in Table 1 apply to calculation of lateral values for single dowel-type fastener connections between wood-based members and connections of wood-based members to steel and concrete/masonry components. Dowel bearing resistance, dowel bearing length, and dowel moment resistance are explicitly considered in the general dowel equations for calculation of lateral connection values. Additional variables that influence lateral connection values include:

- fastener type;
- fastener failure modes (e.g. tension, bearing, and shear);
- fastener spacing, edge, and end distance;
- connection fabrication and tolerances;
- connection geometry;
- multiple fasteners and group action;
- member strength at the connection; and
- adjustments for end use.

Guidelines and recommended practice for the above variables are provided in the NDS for Wood Construction.

1.2.1 Nominal Limit State Values

Nominal lateral connection values at various limit states (see ASTM D 5652, Figure 4) such as proportional limit, 5% offset, and ultimate (maximum) load are obtained by taking the minimum calculated value, $P$, using all yield mode equations in Table 1.

Property estimates of dowel bearing resistance and dowel moment resistance should be based on the limit state being investigated. In this Technical Report, proportional limit load is the load at which the load-deformation curve deviates from a straight line fitted to the initial linear portion of the load-deformation curve. The 5% offset load is the load at which the load-deformation curve intersects a line represented by the initial tangent modulus offset 5 percent of the fastener diameter. The 5% offset load is intermediate between proportional limit and ultimate loads and represents the nominal yield value for dowel bearing strengths and fastener bending strengths in the NDS. The ultimate load limit state is synonymous with the limit state at maximum load. Note that failure load, which is typically less than ultimate load and which generally occurs after ultimate load has been reached, is not a limit state that can be modeled by the general dowel equations.

The term “nominal” is used to designate values that have not been modified by design “adjustment” factors, such as load duration, wet service, etc., as shown in NDS Table 7.3.1.

1.2.2 Nominal Design Value

The nominal design value, $Z$, is the minimum of the calculated 5% offset strength, $P_{5\%}$, for all yield
mode equations in Table 1 divided by applicable reduction terms in Table 2.

In the NDS, reduction terms are integral with the yield limit equations and appear in the denominator of each of the available yield mode equations (See 1.2.2.1 and Table 2). These terms adjust nominal yield values (based on the 5% offset yield point) to values representative of nominal proportional limit based design values of early editions of the Specification. In this Technical Report, reduction terms and calculation of yield mode values are separated to permit direct calculation of limit state values of proportional limit, 5% offset and ultimate load. When using this Technical Report to calculate nominal lateral design values from the NDS, 5% offset property estimates of dowel bearing resistance and fastener bending yield strength should be used. Note that fastener bending yield strengths and dowel bearing strengths appearing in the NDS are based on 5% offset property estimates and are identical to 5% offset property estimates presented in this report.

Nominal design values, \( Z \), are identical to NDS yield limit values for bolt, nail and spike connections provided equivalent input parameters are used (e.g. dowel bearing resistance, dowel moment resistance, and dowel bearing length). For lag screw and wood screw connections, identical values result when the dowel moment resistance in the main member is set to 75% of the dowel moment resistance in the side member (\( M_m = 0.75 M_s \)). Nominal design values for drift bolt and drift pin connections equal 75% of the nominal design value for bolts of the same diameter.

1.2.2.1 Reduction Terms for \( P_{5\%} \) Values

Reduction terms in Table 2, which adjust \( P_{5\%} \) values to nominal design values are identical to those used in the NDS yield limit equations. Table 2 also provides guidance on reduction terms for yield modes not covered in the NDS due to simplifying assumptions used to develop the NDS yield limit equations. Reduction terms adjusting ultimate or proportional limit values to nominal design values have not been established. Proportional limit and ultimate values of fastener bending strength and dowel bearing strength, however, are provided (see 1.3.1 and 1.3.2) to demonstrate the applicability of the general dowel equations at various limit states.

1.3 Input Parameters

Basic information needed to calculate lateral connection values includes dowel-bearing resistance, dowel moment resistance, dowel bearing length, and gap distance.

1.3.1 Dowel bearing resistance

Dowel diameter and dowel bearing strength are used to determine dowel bearing resistance, \( q_s \) and \( q_m \), in the side and main member respectively. For wood-based members, dowel bearing strength estimates based on dowel bearing strength of the material should be used and are generally based on test methods outlined in ASTM D 5764. Table 3 provides guidance for estimating dowel bearing strengths. Table 4 contains specific dowel bearing strengths including 5% offset dowel-bearing strengths assumed for tabulated NDS connection values.

1.3.2 Dowel moment resistance

Dowel bending strength and applicable section modulus are used to determine dowel moment resistance, \( M_s \) and \( M_m \), in the side and main member respectively. Dowel bending strengths are generally based on test methods outlined in ASTM F 1575, estimates of dowel tensile strength as determined by ASTM F 606, or tabulated tensile values for the dowel material. A guide for estimating bending strength is outlined in Table 5. Table 6 contains specific dowel bending strengths including 5% offset dowel bending strengths assumed for tabulated NDS connection values.

1.3.3 Dowel bearing length

Dowel bearing lengths, \( \ell_s \) and \( \ell_m \), represent the length of dowel bearing in the side and main member, respectively. For double shear connections, the minimum length of bearing in either of the side members should be used. The tapered tip of a lag screw should not be included in determination of bearing length (See NDS
Appendix L for typical lag screw dimensions). For most wood-to-wood and wood-to-metal bolted connections, where the longitudinal axis of the bolt is perpendicular to the faces of connected members, the dowel bearing length in the side and main member equals the side and main member thickness respectively. For wood-to-concrete connections, embedment length in the concrete should be used as the dowel bearing length in the concrete member.

1.3.4 Gap distance

Gap distance, \( g \), is the distance measured between adjacent faces of connected members. Gap distance equals zero for connected members with adjacent faces in contact.

1.4 Other Considerations

1.4.1 Penetration Effects

The NDS penetration depth factor is applicable when lateral capacity is based on checking yield modes considered in the NDS for nail, spike, wood screw and lag screw connections only. It conservatively reduces available NDS yield mode values to account for yield modes not explicitly addressed.

When checking all six yield modes using the general dowel equations, it is not necessary to reduce calculated lateral values by the NDS penetration depth factor, \( C_p \). To do so would yield very conservative design values, since the effect of main member bearing length on connection capacity is rationally addressed by considering all connection yield modes (See Figure 1).

When nail, spike or wood screw penetration, including tip, is reduced below 10D (10 fastener diameters) the impact of neglecting the reduction in bearing capacity due to the reduced fastener tip diameter can become significant. For penetration less than 10D, the user should exclude the fastener tip length from their assumed dowel bearing length (Note: Tip lengths of diamond point nails, such as common and box nails, range from approximately 1.3 to 2.0 nail diameters in length). In all cases, limiting minimum penetration depths in accordance with NDS minimums (e.g. 6D for nails and spikes, 4D for wood screws and lag screws) should be maintained.

1.4.2 End Fixity

End fixity refers to resistance to rotation provided at the end(s) of the dowel. This action is not specifically addressed in the derivation of the NDS yield limit equations nor in the general dowel equations. Contribution of end fixity is dependant on several factors including load level, fastener type and installation, washer or fastener head size (e.g. nail or screw head size), and amount of dimensional change in connected members. Single shear connections exhibiting yield mode II, III, or III\(_m\), shown in Figure 1, are prone to be influenced by end fixity. For double shear, connections exhibiting mode III behavior are prone to be influenced by end fixity. These yield modes are prone to be influenced by end fixity because they are limited by dowel bearing resistance as the dowel rotates. The moment resistance contributed by end fixity tends to force fastener yielding in a connection otherwise controlled by wood bearing. In all cases, it is necessary for the washer or fastener head to maintain contact with the member in order to realize the contribution of end fixity. As a result, consideration of dimensional changes resulting from wood shrinkage, influence of load level, wood bearing beneath the washer or fastener head, and fastener type should be addressed before accounting for end fixity in design. End fixity is conservatively ignored in the development of general dowel equations and the NDS yield limit equations.

1.4.3 Friction

The effect of friction between members is largely dependant on type of fastener, condition of wood, amount of shrinkage, and relaxation of the wood member. Friction is usually not accounted for in wood connection design because the amount of frictional force is difficult to predict and in many instances may not exist as wood shrinks or the connection relaxes. Frictional resistance to slipping of connection members is conservatively ignored in the development of the general dowel equations and the NDS yield limit equations.
1.5 Yield Mode Actual Versus Predicted

The general dowel equations and the NDS yield limit equations assume that the connection yields in accordance with one of the yield modes depicted in Figure 1 and as described in Part III - Equation Derivation. Predicted yield modes at various limit states up to ultimate load are in agreement with yield modes observed from connection tests. It should be noted that predicted yield modes for single fastener connections with sufficient end and edge distances are indicative of connection yielding up to ultimate load but do not represent damage observed at failure. Wood splitting along a line of fasteners, formation of a shear plug, fastener shear, and fastener withdrawal are typical failures observed in connections tested beyond ultimate load.
Figure 1  Connection Yield Modes

Single Shear Connections

Mode I \textsubscript{m}

Mode I \textsubscript{s}

Mode II

Mode III \textsubscript{m}

Mode III \textsubscript{s}

Mode IV

Double Shear Connections
Table 1  General Dowel Equations

<table>
<thead>
<tr>
<th>Yield Mode</th>
<th>Single Shear</th>
<th>Double Shear</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I&lt;sub&gt;m&lt;/sub&gt;</td>
<td>( P = q_m \ell_m )</td>
<td>( P = q_m \ell_m )</td>
<td>Main member bearing</td>
</tr>
<tr>
<td>I&lt;sub&gt;s&lt;/sub&gt;</td>
<td>( P = q_s \ell_s )</td>
<td>( P = 2q_s \ell_s )</td>
<td>Side member bearing</td>
</tr>
<tr>
<td>II-IV</td>
<td>( P = \frac{-B + \sqrt{B^2 - 4AC}}{2A} )</td>
<td>( P = \frac{-B + \sqrt{B^2 - 4AC}}{A} )</td>
<td>General equation for member bearing and dowel yielding</td>
</tr>
</tbody>
</table>

Inputs A, B, & C for Yield Modes II-IV

\[
\begin{align*}
II^1 & \quad A = \frac{1}{4q_s} + \frac{1}{4q_m} \\
III_{m,1,2}^1 & \quad A = \frac{1}{2q_s} + \frac{1}{4q_m} \\
III_{s,2}^2 & \quad A = \frac{1}{4q_s} + \frac{1}{2q_m} \\
IV_{m,2}^2 & \quad A = \frac{1}{2q_s} + \frac{1}{2q_m}
\end{align*}
\]

Notes:
- \( P \) = nominal lateral connection value, lbs.
- \( g \) = gap between members, in.
- \( \ell_m \) = main member dowel bearing length, in.
- \( \ell_s \) = side member dowel bearing length, in.
- \( F_b \) = dowel bending strength, psi
- \( q_s \) = side member dowel-bearing resistance = \( F_b D \), lbs./in.
- \( q_m \) = main member dowel-bearing resistance = \( F_m D \), lbs./in.
- \( D_s \) = dowel diameter at max. stress in side member, in.
- \( D_m \) = dowel diameter at max. stress in main member, in.
- \( F_{bs} \) = side member dowel-bearing strength, psi
- \( M_s \) = side member dowel moment resistance<sup>2</sup>, in-lbs. = \( F_b (D /6) \)
- \( F_{ms} \) = main member dowel-bearing strength, psi
- \( M_m \) = main member dowel moment resistance<sup>2</sup>, in-lbs. = \( F_m (D /6) \)

<sup>1</sup>Yield Modes II and III<sub>m</sub> are not applicable to double shear connections.

<sup>2</sup>For proportional limit values, \( M_s = F_{bs}(\pi D /32) \) and \( M_m = F_{bs}(\pi D_m /32) \).

Table 2  Reduction Terms Adjusting \( P_{5\%} \) Values to Nominal Design Values

<table>
<thead>
<tr>
<th>Fastener Type</th>
<th>Yield Mode</th>
<th>Reduction Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts, drift pins</td>
<td>I&lt;sub&gt;m&lt;/sub&gt;, I&lt;sub&gt;s&lt;/sub&gt;</td>
<td>4 ( K_0 )</td>
</tr>
<tr>
<td>\</td>
<td>II</td>
<td>3.6 ( K_0 )</td>
</tr>
<tr>
<td>\</td>
<td>III&lt;sub&gt;m&lt;/sub&gt;, III&lt;sub&gt;s&lt;/sub&gt;, IV</td>
<td>3.2 ( K_0 )</td>
</tr>
<tr>
<td>Lag screws</td>
<td>I&lt;sub&gt;m&lt;/sub&gt;, I&lt;sub&gt;s&lt;/sub&gt;</td>
<td>4 ( K_0 )</td>
</tr>
<tr>
<td>\</td>
<td>II&lt;sup&gt;1&lt;/sup&gt;, II&lt;sup&gt;1&lt;/sup&gt;, III&lt;sub&gt;m&lt;/sub&gt;, \</td>
<td>2.8 ( K_0 )</td>
</tr>
<tr>
<td>\</td>
<td>III&lt;sub&gt;s&lt;/sub&gt;, IV</td>
<td>3 ( K_0 )</td>
</tr>
<tr>
<td>Nails, spikes</td>
<td>I&lt;sub&gt;m&lt;/sub&gt;, I&lt;sub&gt;s&lt;/sub&gt;</td>
<td>\</td>
</tr>
<tr>
<td>\</td>
<td>II&lt;sup&gt;1&lt;/sup&gt;, II&lt;sup&gt;1&lt;/sup&gt;, III&lt;sub&gt;m&lt;/sub&gt;, III&lt;sub&gt;s&lt;/sub&gt;, IV</td>
<td>\</td>
</tr>
<tr>
<td>Wood screws</td>
<td>I&lt;sub&gt;m&lt;/sub&gt;, I&lt;sub&gt;s&lt;/sub&gt;</td>
<td>\</td>
</tr>
<tr>
<td>\</td>
<td>II&lt;sup&gt;1&lt;/sup&gt;, II&lt;sup&gt;1&lt;/sup&gt;, III&lt;sub&gt;m&lt;/sub&gt;, III&lt;sub&gt;s&lt;/sub&gt;, IV</td>
<td>\</td>
</tr>
</tbody>
</table>

Notes:
- \( K_0 = 1 + 0.25(\theta /90) \)
- \( \theta \) = maximum angle of load to grain (0° ≤ \( \theta \) ≤ 90°)
- \( K_0 = 2.2 \) for \( D \leq 0.17" \)
- \( K_0 = 10D + 0.5 \) for \( 0.17" < D < 0.25" \)
- \( K_0 = 3.0 \) for \( D \geq 0.25" \)

<sup>1</sup>Yield modes and corresponding reduction terms are not provided in the NDS.
### Table 3 Dowel Bearing Strength Estimates, $F_e$

<table>
<thead>
<tr>
<th>Material</th>
<th>Proportional limit, $F_{e,pl}$</th>
<th>5% Offset, $F_{e,5%}$</th>
<th>Ultimate, $F_{e,ult}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood-based</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallel-to-grain</td>
<td>0.67 $F_{e,ult}$</td>
<td>$F_{e,5%}$</td>
<td>$F_{e,ult}$</td>
</tr>
<tr>
<td>perpendicular-to-grain</td>
<td>0.50 $F_{e,ult}$</td>
<td>$F_{e,5%}$</td>
<td>$F_{e,ult}$</td>
</tr>
<tr>
<td>Metal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.024'' \leq t &lt; 0.1875''$</td>
<td>0.67 $F_u$</td>
<td>$F_u$</td>
<td>$3F_u$</td>
</tr>
<tr>
<td>$t \geq 3/16''$</td>
<td>0.67 $F_u$</td>
<td>$F_u$</td>
<td>$1.5F_u$</td>
</tr>
<tr>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5 $f'_c$</td>
<td>3$f'_c$</td>
<td>5$f'_c$</td>
</tr>
</tbody>
</table>

Notes:
- $t = $ thickness, in.
- $f'_c = $ compressive strength, psi
- $F_{u} = $ tensile strength, psi

15% offset and ultimate dowel bearing strengths for wood-based products are based on test methods outlined in ASTM D 5764.

2$F_{e,ult}$ based on AISC (1989) and AISI (1996).

3$F_{e,ult}$ based on Vintzeleou (1986). Experimental verification of $F_{e,ult}$ (Biolzi, 1990) based on concrete compressive strength, $f'_c$, of 2700 psi. $F_{e,pl}$, $F_{e,5%}$, and $F_{e,ult}$ estimates should be limited to concrete compressive strengths of 2700 psi or less.

### Table 4 Dowel Bearing Strengths, $F_e$, for Various Connection Materials (Tabulated NDS Connection Values Are Based on $F_{e,5%}$) 1,4

<table>
<thead>
<tr>
<th>Material</th>
<th>$F_{e,pl}$ (psi)</th>
<th>$F_{e,5%}$ (psi)</th>
<th>$F_{e,ult}$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumber (bolt, drift pin, lag screw)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallel-to-grain</td>
<td>7862G^{1.07/D^{0.17}}</td>
<td>11200G</td>
<td>11735G^{1.07/D^{0.17}}</td>
</tr>
<tr>
<td>perpendicular-to-grain</td>
<td>3178G^{1.15/D^{0.51}}</td>
<td>6100G^{1.45/D^{0.5}}</td>
<td>6355G^{1.15/D^{0.51}}</td>
</tr>
<tr>
<td>Lumber (nail, wood screw)</td>
<td></td>
<td>16600G^{1.84}</td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASTM A653 Grade 33 (0.036”&lt;t&lt;3/16”)</td>
<td>30,150</td>
<td>45,000</td>
<td>135,000</td>
</tr>
<tr>
<td>ASTM A36 (t&gt;3/16”)</td>
<td>38,860</td>
<td>58,000</td>
<td>87,000</td>
</tr>
<tr>
<td>Concrete ($f'_c = 2000$ psi)</td>
<td>5,000</td>
<td>6,000</td>
<td>10,000</td>
</tr>
</tbody>
</table>

Notes:
- $G = $ specific gravity (Oven dry weight and volume)
- $f'_c = $ compressive strength, psi
- $D = $ fastener shank diameter, in
- $t = $ thickness, in

1Calculation of connection values using NDS yield limit equations or general dowel equations is not limited to materials or dowel bearing strength values provided in this table.

2$F_{e,5%}$ for lumber based on Wilkinson (1991). Data from Wilkinson report on dowel bearing strength was used to derive other $F_e$ values for lumber.

3$F_{e,pl}$ and $F_{e,ult}$ for nails and spikes are 80% of the values for bolts of equivalent diameter.

4See Additional References for information on dowel bearing strength, $F_e$, of panel products.
### Table 5 Dowel Bending Strength Estimates, $F_b$

<table>
<thead>
<tr>
<th>Fastener Type</th>
<th>Proportional limit, $F_{b,pl}$</th>
<th>5% Offset, $F_{b,5%}$</th>
<th>Ultimate, $F_{b,ult}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolts, lag screws, drift pins</td>
<td>$F_y$</td>
<td>$F_y/2 + F_y/2$</td>
<td>$F_u$</td>
</tr>
<tr>
<td>Common nails, box nails, spikes, lag screws (D ≤ 3/8’’), wood screws&lt;sup&gt;1&lt;/sup&gt;</td>
<td>$0.6F_{b,ult}$</td>
<td>$F_{b,5%}$</td>
<td>$F_{b,ult}$</td>
</tr>
</tbody>
</table>

**Notes:**
- $D =$ fastener shank diameter, in.
- $F_y =$ fastener tensile yield strength, psi
- $F_u =$ fastener tensile strength, psi
- $F_{b,ult} =$ dowel bending strength, psi

<sup>1</sup>Bending strengths are generally based on fastener bending tests outlined in ASTM F 1575.

### Table 6 Bending Strengths, $F_b$, for Dowel Type Fasteners (Tabulated NDS Connection Values Are Based On $F_{b,5%}$)<sup>1</sup>

<table>
<thead>
<tr>
<th>Fastener Type</th>
<th>$F_{b,pl}$ (psi)</th>
<th>$F_{b,5%}$ (psi)</th>
<th>$F_{b,ult}$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt, lag screw, drift pin&lt;sup&gt;2&lt;/sup&gt;</td>
<td>36,000</td>
<td>45,000</td>
<td>60,000</td>
</tr>
<tr>
<td>Common nail, box nail, spike, lag screw, wood screws&lt;sup&gt;3&lt;/sup&gt;</td>
<td>36,000</td>
<td>45,000</td>
<td>60,000</td>
</tr>
<tr>
<td>0.099 ≤ $D$ ≤ 0.142</td>
<td>78,000</td>
<td>100,000</td>
<td>130,000</td>
</tr>
<tr>
<td>0.142 &lt; $D$ ≤ 0.177</td>
<td>69,000</td>
<td>90,000</td>
<td>115,000</td>
</tr>
<tr>
<td>0.177 &lt; $D$ ≤ 0.244</td>
<td>54,000</td>
<td>80,000</td>
<td>90,000</td>
</tr>
<tr>
<td>0.244 &lt; $D$ ≤ 0.273</td>
<td>48,000</td>
<td>70,000</td>
<td>80,000</td>
</tr>
<tr>
<td>0.273 &lt; $D$ ≤ 0.344</td>
<td>42,000</td>
<td>60,000</td>
<td>70,000</td>
</tr>
<tr>
<td>0.344 &lt; $D$ ≤ 0.375</td>
<td>36,000</td>
<td>45,000</td>
<td>60,000</td>
</tr>
<tr>
<td>Threaded hardened steel nail&lt;sup&gt;4&lt;/sup&gt;</td>
<td>-</td>
<td>130,000</td>
<td>-</td>
</tr>
<tr>
<td>0.120 ≤ $D$ ≤ 0.142</td>
<td>-</td>
<td>115,000</td>
<td>-</td>
</tr>
<tr>
<td>0.142 &lt; $D$ ≤ 0.192</td>
<td>-</td>
<td>100,000</td>
<td>-</td>
</tr>
<tr>
<td>0.192 &lt; $D$ ≤ 0.207</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Notes:**
- $D =$ fastener shank diameter, in.
- $F_{b,pl} =$ dowel bending strength, psi

<sup>1</sup>Calculation of connection values using NDS yield limit equations or general dowel equations are not limited to fastener bending strength values provided in this table.

<sup>2</sup>Bolt, lag screw, drift pin - SAE J429-Grade 1 with $F_{y,\text{min}} =$ 36,000 psi and $F_{u,\text{min}} =$ 60,000 psi. Use of the dowel bending strength estimate outlined in Table 5 results in a $F_{b,5%}$ equal to 48,000 psi, however, 45,000 psi has been assumed for consistency with the bending strength assumption used for tabulated values in the NDS.

<sup>3</sup>Common nail, box nail, spike, wood screw, lag screw (D ≤ 3/8”) with low to medium carbon steel. See Loferski (1991) for bending yield strength of nails.

<sup>4</sup>Threaded hardened steel nail with medium carbon steel.
Example problems are based on the application of the general dowel equations to single fastener connections. Connection values for all yield modes are provided for each example and minimum values for each configuration are underlined. Connection values have not been adjusted for conditions of end use such as load duration, wet service, and temperature. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

For each example problem, connection values in parenthesis represent yield mode values calculated using NDS yield limit equations. Note that NDS connection provisions are not directly applicable for several of the conditions and limit states covered in the Example problems and that certain yield modes are excluded from consideration. For cases where NDS connection provisions apply, values from the general dowel equations and NDS yield limit equations are identical provided equivalent inputs are used.

Example 2.1 - Bolted Connection With Gap

**Problem Statement:** Determine the nominal design value for a single shear bolted connection between sawn lumber members. Compare the values for gap distance \( g \) equal to 0 inch, \( \frac{1}{4} \) inch, and \( \frac{1}{2} \) inch.

**Given:** The parallel and perpendicular to grain dowel bearing strengths are equal to 4800 psi and 2550 psi, respectively. Both side and main member thicknesses are 1-1/2 inches. Each connection uses a single \( \frac{1}{2} \) inch diameter bolt (SAE J429 Grade 1), and the load is applied parallel and perpendicular to grain.

\[
\ell_i, \ell_m = 1.5 \text{ inches} \\
D_s, D_m = 0.5 \text{ inch} \\
F_{b,5\%} = 45,000 \text{ psi} \\
F_{c,s} = 4800 \text{ psi}, F_{c,e} = 2550 \text{ psi} \\
g = 0, 0.25, 0.5 \text{ inch}
\]

<table>
<thead>
<tr>
<th>Bearing length, in. &amp; grain direction</th>
<th>Yield mode value, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main</td>
<td>Side</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>Gap distance, ( g = 0 ) in.</td>
<td></td>
</tr>
<tr>
<td>1-1/2</td>
<td>1-1/2</td>
</tr>
<tr>
<td>1-1/2</td>
<td>1-1/2_</td>
</tr>
<tr>
<td>1-1/2_</td>
<td>1-1/2 _</td>
</tr>
<tr>
<td>Gap distance, ( g = \frac{1}{4} ) in.</td>
<td></td>
</tr>
<tr>
<td>1-1/2</td>
<td>1-1/2</td>
</tr>
<tr>
<td>1-1/2</td>
<td>1-1/2_</td>
</tr>
<tr>
<td>1-1/2_</td>
<td>1-1/2 _</td>
</tr>
<tr>
<td>Gap distance, ( g = \frac{1}{2} ) in.</td>
<td></td>
</tr>
<tr>
<td>1-1/2</td>
<td>1-1/2</td>
</tr>
<tr>
<td>1-1/2</td>
<td>1-1/2_</td>
</tr>
<tr>
<td>1-1/2_</td>
<td>1-1/2 _</td>
</tr>
</tbody>
</table>

Underlined values represent nominal design values. Connection values in parentheses represent yield mode values calculated using the NDS yield limit equations.
**Example 2.2 - Lag Screw Connection**

**Problem Statement:** Determine the nominal design value for a single shear lag screw connection between sawn lumber members. Compare values assuming that fastener moment resistances in the side and main member are:

1) equal and based on the fastener root diameter, \( D \) (e.g. connection with fully threaded lag screw);

2) unequal with fastener moment resistance in the main member equal to 75\% of the fastener moment resistance in the side member, \( M_m = 0.75 M_s \) where \( M_s \) is based on the unthreaded shank diameter, \( D \) (e.g. connection with unthreaded shank slightly extended into the main member); and

3) equal and based on the unthreaded shank diameter, \( D \) (e.g. connection with unthreaded shank extending deep into the main member).

Consider side member loading parallel and perpendicular to grain and main member loading parallel to grain. Case 2 handling of fastener moment resistance is consistent with the treatment of fastener moment resistance in the NDS yield limit equations for lag screws and wood screws.

**Given:** The parallel and perpendicular to grain dowel bearing strengths are equal to 6150 psi and 2950 psi, respectively. Side member thickness is 2-1/2 inches and lag screw bearing length in the main member is 6 inches. Each connection uses a single 3/4 inch diameter lag screw.

\[ \begin{align*}
\ell_s &= 2.5 \text{ inches}, \quad \ell_m = 6 \text{ inches} \\
D &= 0.75 \text{ inch}, \quad D_r = 0.579 \text{ inch} \\
F_{b,3\%} &= 45,000 \text{ psi} \\
F_{c,\text{p}} &= 6150 \text{ psi}, \quad F_{c,\text{e}} = 2950 \text{ psi} \\
g &= 0 \text{ inch}
\end{align*} \]

<table>
<thead>
<tr>
<th>Bearing length, in. &amp; grain orientation</th>
<th>Yield mode value, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( I_m )</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
</tr>
<tr>
<td>2-(\frac{1}{2}),</td>
<td>6919</td>
</tr>
<tr>
<td>2-(\frac{1}{2})(_\perp)</td>
<td>5535</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
</tr>
<tr>
<td>2-(\frac{1}{2}),</td>
<td>6919</td>
</tr>
<tr>
<td>2-(\frac{1}{2})(_\perp)</td>
<td>5535</td>
</tr>
<tr>
<td><strong>Case 3</strong></td>
<td></td>
</tr>
<tr>
<td>2-(\frac{1}{2}),</td>
<td>6919</td>
</tr>
<tr>
<td>2-(\frac{1}{2})(_\perp)</td>
<td>5535</td>
</tr>
</tbody>
</table>

Underlined values represent nominal design values. Connection values in parentheses represent yield mode values calculated using the NDS yield limit equations.
Example 2.3 - Nailed Connection

**Problem Statement:** Determine the nominal design, proportional limit, 5% offset, and ultimate values for a single shear nailed connection between sawn lumber members.

**Given:** Proportional limit; 5% offset; and ultimate dowel bearing strength are equal to 4088 psi, 4637 psi, and 6093 psi respectively. The side member thickness is 1-1/2 inches and penetration into the main member is 2 inches (p = 2.0 inches). A single 16d (0.162 inch diameter x 3-1/2" long) common nail is used in the connection. The load direction for both side and main members is parallel to grain.

\[ \ell_s = 1.5 \text{ inches} \]
\[ \ell_m = 2.0 \text{ inches} \]
\[ D_s, D_m = 0.162 \text{ inch} \]
\[ F_{c,pl} = 4083 \text{ psi}, \quad F_{b,pl} = 69,000 \text{ psi} \]
\[ F_{c,5\%} = 4637 \text{ psi}, \quad F_{b,5\%} = 90,000 \text{ psi} \]
\[ F_{c,ult} = 6093 \text{ psi}, \quad F_{b,ult} = 115,000 \text{ psi} \]
\[ g = 0 \text{ inch} \]

<table>
<thead>
<tr>
<th>Nail Size</th>
<th>Yield mode, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I_m</td>
</tr>
<tr>
<td>16d common (0.162&quot; dia. x 3 1/2&quot;)</td>
<td>512</td>
</tr>
</tbody>
</table>

Underlined values represent nominal values for the given limit state. Connection values in parentheses represent yield mode values calculated using the NDS yield limit equations.

Example 2.4 - Nailed Connection With Reduced Penetration

**Problem Statement:** Determine the nominal design value for a single shear nailed connection between sawn lumber members.

**Given:** Side and main member thickness equals 1-1/2 inches, and have a dowel bearing strength equal to 4637 psi. A single 16d (0.162 inch diameter x 3-1/2” long) common nail is used to make the connection.

\[ \ell_s = 1.5 \text{ inches}, \quad \ell_m = 1.5 \text{ inches} \]
\[ D_s, D_m = 0.162 \text{ inch} \]
\[ F_{c,5\%} = 4637 \text{ psi}, \quad F_{b,5\%} = 90,000 \text{ psi} \]
\[ g = 0 \text{ inch} \]

<table>
<thead>
<tr>
<th>Nail Size</th>
<th>Yield mode, lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I_m</td>
</tr>
<tr>
<td>16d common (0.162&quot; dia. x 3 1/2&quot;)</td>
<td>512</td>
</tr>
</tbody>
</table>

The underlined value of 141 lbs represents the nominal design value calculated in accordance with the general dowel equations and recommendations for treatment of fastener bearing length in the main member. Connection values in parentheses represent yield mode values calculated using the NDS yield limit equations multiplied by the NDS Penetration Depth Factor, \( C_d \), of 0.77.
PART III. EQUATION DERIVATION

The yield model used to develop the general dowel equations considers effects of dowel moment resistance and dowel bearing resistance on a connection’s lateral strength. Based on the European Yield Model (Solits 1991), connection strength is assumed to be reached when: (1) compressive strength of the member beneath the dowel is exceeded; or (2) one or more plastic hinges forms in the dowel. Behavior of the connection is assumed to be in accordance with yield modes depicted in Figure 1. Dowel loading is assumed to be uniformly distributed and perpendicular to the axis of the dowel (e.g. ideally plastic deformation). Effects of end fixity, tension forces in the fastener, and friction between members is ignored.

Each yield mode addresses a specific loading condition on the dowel such that the dowel will remain in static equilibrium. General dowel equations can be obtained by considering equilibrium of forces within a connection exhibiting behavior in accordance with yield modes I - IV. Applying this concept, a free-body diagram for each yield mode can be drawn, and principles of statics can be used to develop the general dowel equations.

**Mode I**

Yield modes I_s and I_m model connections limited by uniform bearing in the main and side member(s). Figure 2 Case A, shows that maximum load, P, is determined by the following equations:

\[
P = q_s \ell \text{ (for side member)}
\]

\[
P = q_m \ell \text{ (for main member)}
\]

Similarly, considering the geometry of a double shear connection, maximum load, P, is determined by:

\[
P = q_m \ell_m
\]

\[
P = 2q_s \ell_s
\]

where,

\[
q_s = \text{side member dowel bearing resistance} = F_{es} D, \text{ lbs./in.}
\]

\[
q_m = \text{main member dowel bearing resistance} = F_{em} D, \text{ lbs./in.}
\]

\[
\ell_s = \text{side member bearing length, in.}
\]

\[
\ell_m = \text{side member bearing length, in.}
\]

\[
F_{es} = \text{side member dowel bearing strength, psi}
\]

\[
F_{em} = \text{main member dowel bearing strength, psi}
\]

\[
D = \text{dowel shank diameter, in.}
\]

**Modes II-IV**

For modes II-IV, consideration of conditions where limiting or maximum moments act on the dowel simplifies equation derivation. These basic conditions are shown in Figure 2. In Case B, maximum moment is based on dowel bearing. In Case C, maximum moment is based on dowel bending. Note that maximum moments for both cases occur at points of zero shear.

Maximum moment due to dowel bearing, shown in Case B of Figure 2, represents a load condition where the dowel is sufficiently large to prevent yielding of the dowel (dowel bending). For calculation purposes, let q define member bearing resistance (lb/in) determined by the equation q = F_D, \ell define member bearing length (in), and let x represent the location of zero shear (x = (\ell-2a) / 2). The resulting maximum moment due to dowel bearing, M_b, equals qa^2. Recognizing that \( a = (\ell-x)/2 \) and \( x = P/q \) and substituting results in M_b = (q/4)(\ell_P/q)^2.

Subscripts s and m indicate side and main member in the following equations for maximum moment due to dowel bearing:

**Single Shear:**

\[
M_{bs} = \frac{q_s \ell_s}{4} \left( \frac{P}{q_s} \right)^2
\]

**Double Shear:**

\[
M_{bs} = \frac{q_s \ell_s}{4} \left( \frac{P}{2q_s} \right)^2
\]
Maximum moment due to dowel bending, is shown in Case C of Figure 2. In this case, maximum moment is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear \((x = P/q)\). Moment resistance of the dowel assuming ideally plastic behavior is expressed as follows:

\[
M_i = \frac{F_b D_s^3}{6}
\]

\[
M_m = \frac{F_{b,pl} D_m^3}{6}
\]

If elastic behavior is assumed, such as when estimating values at proportional limit, the moment resistance of the dowel can be expressed as follows:

\[
M_i = \frac{\pi F_{b,pl} D_s^3}{3 / 2}
\]

\[
M_m = \frac{\pi F_{b,pl} D_m^3}{3 / 2}
\]

Assumed loading conditions for each of the yield modes I-IV is provided in Figure 3. Each mode consists of an interaction of dowel bearing and dowel bending as shown in Figure 2. Considering equilibrium of the dowel for each particular yield mode (by summing moments about a fixed point on the dowel), and using relationships for maximum moment defined above, characteristic equations for the maximum load, \(P\), can be determined as follows:

**Single Shear:**

\[
P^2 \left(\frac{1}{2d_i} + \frac{1}{2d_m}\right) = P g - (M_{ls} + M_{lm}) = 0
\]

**Figure 2 General Conditions of Dowel Loading**

- **Case A**
  - Moment, \(M\)
  - Shear, \(V\)
  - \(P = V = qI\)
  - \(M = qI / 2\)

- **Case B**
  - Moment, \(M\)
  - Shear, \(V\)
  - \(P = V = qx\)
  - \(M_{max} = qx\)

- **Case C**
  - Moment, \(M\)
  - Shear, \(V\)
  - \(P = V = qx\)
  - \(M_{max} = M_{dowel}\)
Double Shear:

\[ \frac{P^2}{4} \left( 1 - \frac{1}{2q_s} + \frac{1}{2q_m} \right) \cdot \frac{Pg}{2} \cdot (M_{L_s} + M_{L_m}) = 0 \]

where,

- \( M_{L_s} = \) maximum moment developed in the side member at \( x_s \)
- \( M_{L_m} = \) maximum moment developed in the main member at \( x_m \)
- \( g = \) gap distance (assumed to be equal for double shear connections)

Variables, \( M_{L_s} \) and \( M_{L_m} \), represent values of maximum moment due to dowel bearing or dowel bending depending on the mode being considered. An example single shear connection with assumed loading for yield mode II is provided in Figure 4. Shear and moment diagrams are also provided. As shown in Figure 4, maximum moments, \( M_{L_s} \) and \( M_{L_m} \) for mode II occur at distances \( x_s \) and \( x_m \) from connected faces and are based on moments, \( M_{bs} \) and \( M_{bm} \), due to dowel bearing.

Derivation of the general dowel equations assumes that critical stresses in the dowel occur at locations of maximum induced moment. This is appropriate for dowels having a constant diameter in the side and main member. Dowel diameters in the side and main member, however, do not need to be equal. For connections where dowel diameter is not constant within a member, it is conservative to assume that the least diameter occurs at the location of maximum moment. Alternatively, critical stress in the dowel can be determined by considering the applicable moment and dowel section properties along the length of the dowel.

Yield modes II and III \(_m\) are not possible for double shear connections as the assumed symmetry of double shear connections does not permit these modes to occur.

**Mode II**

Yield Mode II models a connection limited by dowel bearing in the side and main members. The maximum induced moment was previously determined as follows:

\[ \frac{P^2}{4} \left( 1 - \frac{1}{2q_s} + \frac{1}{2q_m} \right) \cdot \frac{Pg}{2} \cdot (M_{L_s} + M_{L_m}) = 0 \]

Single Shear:

\[ M_{bs} = \frac{q_s}{4} \left( \frac{\ell_s}{q_s} - \frac{P}{q_s} \right)^2 \]

\[ M_{bm} = \frac{q_m}{4} \left( \frac{\ell_m}{q_m} - \frac{P}{q_m} \right)^2 \]

Substituting \( M_{bs} \) and \( M_{bm} \) into the characteristic equation for \( M_{ls} \) and \( M_{lm} \) results in the following quadratic equation expressed in terms of known properties of \( q_s \), \( q_m \), \( \ell_s \), \( \ell_m \), and \( g \), and a single unknown variable, \( P \).

**Mode III \(_m\)**

Mode III \(_m\) models a connection limited by dowel bearing in the main member and dowel bending in the side member. The maximum load, \( P \), is determined by solving the characteristic equation for \( P \) where:

Single Shear:

\[ M_s = \text{dowel bending moment in the side member} \]

\[ M_{bs} = \frac{q_s}{4} \left( \frac{\ell_s}{q_s} - \frac{P}{q_s} \right)^2 \]

Substituting \( M_s \) and \( M_{bs} \) into the characteristic equation for \( M_{ls} \) and \( M_{lm} \) results in the following quadratic equation expressed in terms of known properties of \( q_s \), \( q_m \), \( \ell_s \), \( M_s \), and \( g \), and a single unknown variable, \( P \).

Single Shear:

\[ \frac{P^2}{4} \left( 1 - \frac{1}{2q_s} + \frac{1}{2q_m} \right) + \frac{P}{2} \left( g + \frac{\ell_m}{2} \right) - \left( M_s + \frac{q_m \ell_m^2}{4} \right) = 0 \]
Mode III

Mode III models a connection limited by dowel bearing in the side member(s) and dowel bending in the main member. The maximum load, \( P \), is determined by solving the characteristic equation for \( P \) where:

**Single Shear:**

\[
M_{bs} = \frac{q_s}{4} \left( \ell_s \cdot \frac{P}{q_s} \right)^2
\]

**Double Shear:**

\[
M_{bs} = \frac{q_s}{4} \left( \ell_s \cdot \frac{P}{2q_s} \right)^2
\]

\( M_m = \) dowel bending moment in the main member

Substituting \( M_{bs} \) and \( M_m \) into the characteristic equations for \( M_s \) and \( M_m \) results in the following quadratic equation(s) expressed in terms of known properties of \( q_s \), \( q_m \), \( R \), \( M_m \), and \( g \), and a single unknown variable, \( P \).

**Single Shear:**

\[
P^2 \left( \frac{1}{2q_s} \right) + P \ell_s + \left( M_s + \frac{1}{M_m} \right) = 0
\]

**Double Shear:**

\[
P^2 \left( \frac{1}{2q_s} \right) + P \ell_s + \left( M_s + \frac{1}{M_m} \right) = 0
\]

Mode IV

Mode IV models a connection limited by dowel bending in the main and side member(s). The maximum load, \( P \), is determined by solving the characteristic equation for \( P \) where:

\( M_s = \) dowel bending moment in the side member(s)

\( M_m = \) dowel bending moment in the main member

Substituting \( M_s \) and \( M_m \) into the characteristic equations for \( M_s \) and \( M_m \) results in the following quadratic equations expressed in terms of known properties of \( q_s \), \( q_m \), \( \ell_s \), \( M_m \), \( M_m \), and \( g \), and a single unknown variable, \( P \).

**Single Shear:**

\[
P^2 \left( \frac{1}{2q_s} \right) + P \ell_s + \left( M_s + M_m \right) = 0
\]

**Double Shear:**

\[
P^2 \left( \frac{1}{2q_s} \right) + P \ell_s + \left( M_s + M_m \right) = 0
\]
Figure 3 Connection Yield Modes Assumed Loading

Single Shear Connections

Mode I_m

Mode I_s

Mode II

Mode III_m

Mode III_s

Mode IV

Double Shear Connections

Mode I_m

Mode I_s

Mode II

Mode III_m

Mode III_s

Mode IV
**Figure 4 Single Shear Connection, Mode II**

![Diagram of Single Shear Dowel Joint]

**Shear Diagram, V**

**Moment Diagram, M**

General Dowel Equations for Calculating Lateral Connection Values

American Forest & Paper Association
References


Additional References


Karacabeyli, E., Lateral Strength of Bolted Wood-to-Concrete Connections, 1997, Forintek Canada Corp., Western Region, Vancouver, B.C.


American Wood Council
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